Dynamic Stochastic Model for a Single Airport Ground Holding Problem

Avijit Mukherjee
Mark Hansen

University of California at Berkeley

Literature

- Static Integer Programming
  - Many papers on deterministic problem
  - Stochastic problem studied in Richetta and Odoni (1993); Hoffman (1997); Ball et al.(2003)
  - Equity issues addressed by Vossen et al. (2002)

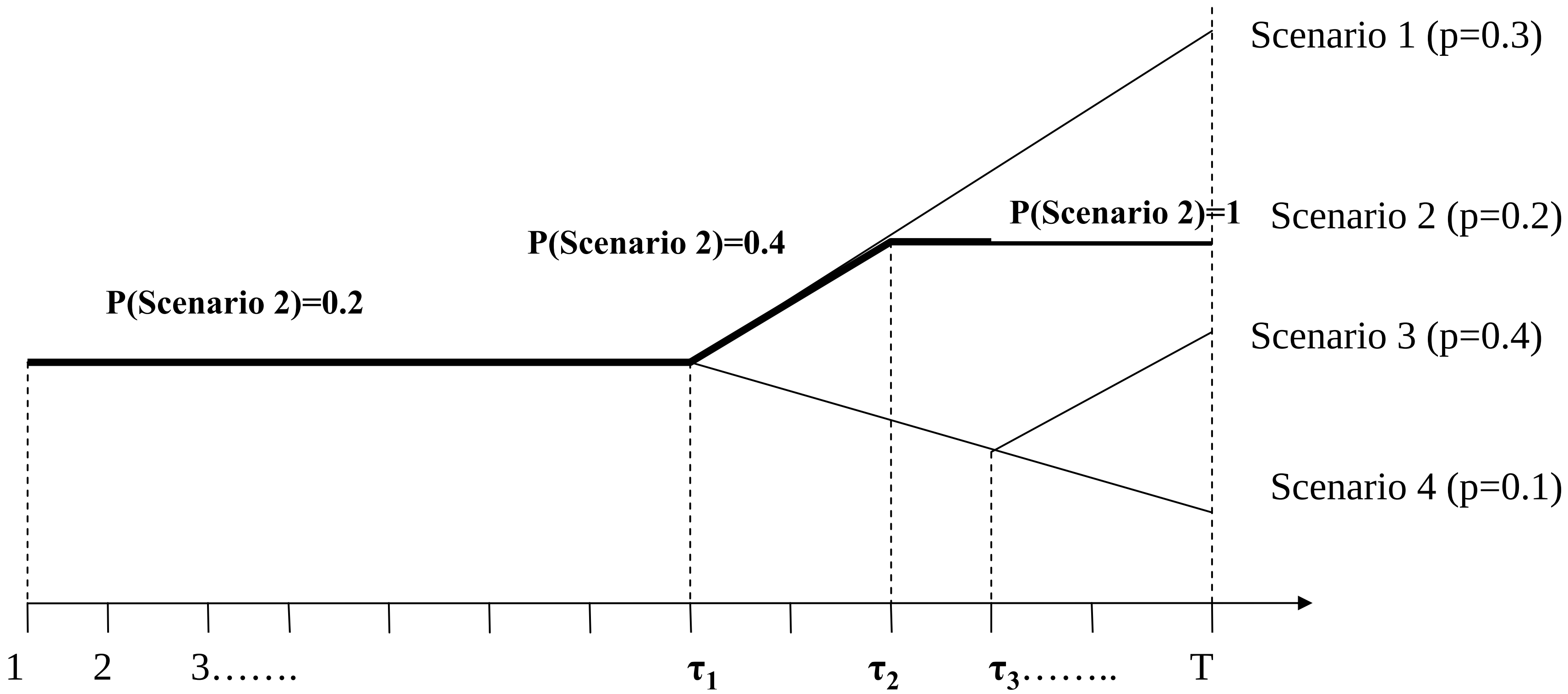
Literature

- Dynamic Models: Richetta and Odoni (1994)
  - Inability to revise previously assigned ground delays
  - Objective function is to minimize expected delay cost
  - No longer Linear Programming Problem if non-linear measure of delay introduced
  - Can handle specific type of scenario tree

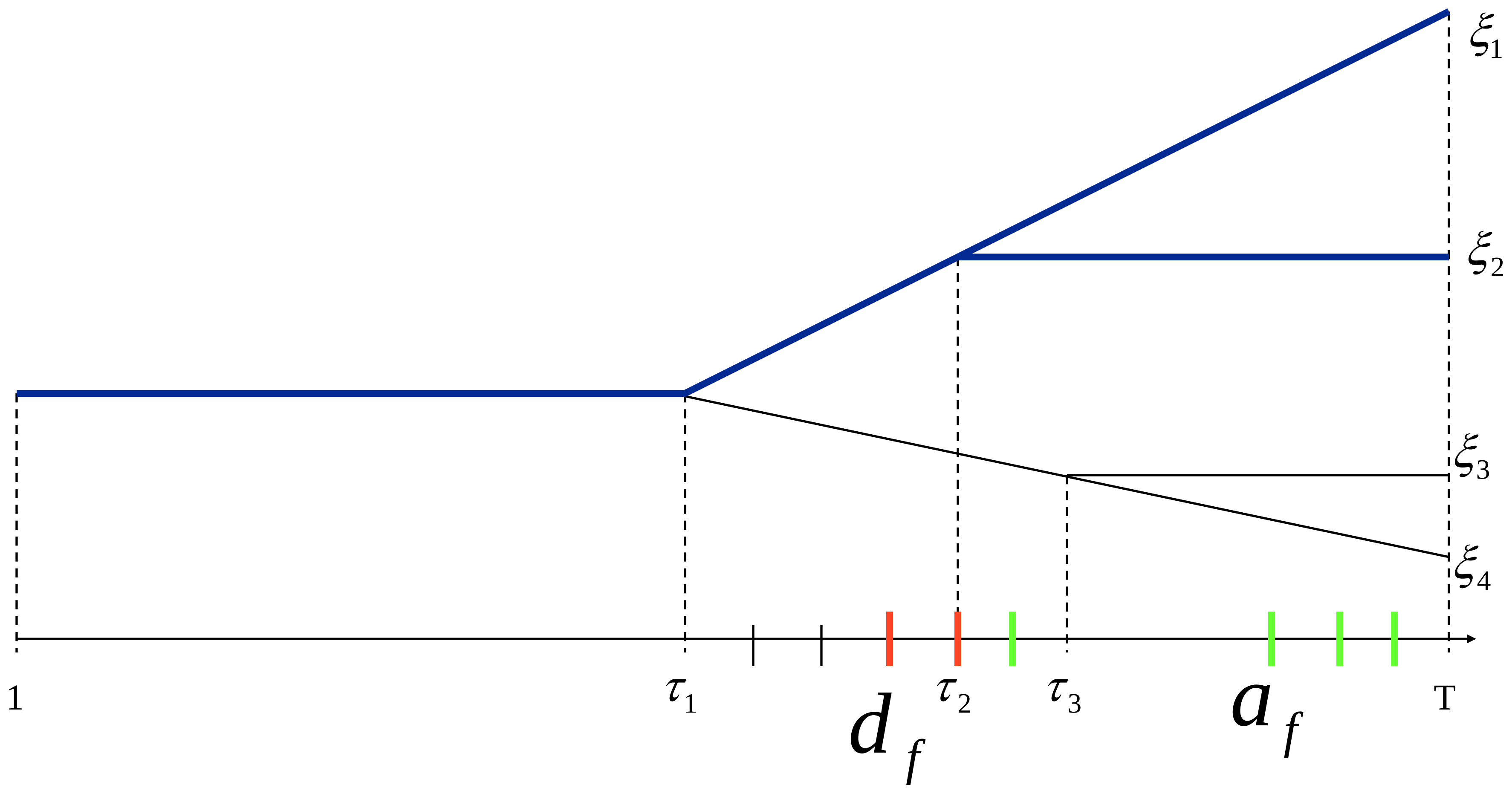
Research Contributions

- Dynamic Stochastic Model for SAGHP
  - Ability to revise ground delays of some flights (non-departed)
  - Can handle any generalized scenario tree
- Alternative Objective Functions
  - Expected Squared Deviation from RBS Allocation
- Multi-Criteria Optimization

Capacity Scenario Tree

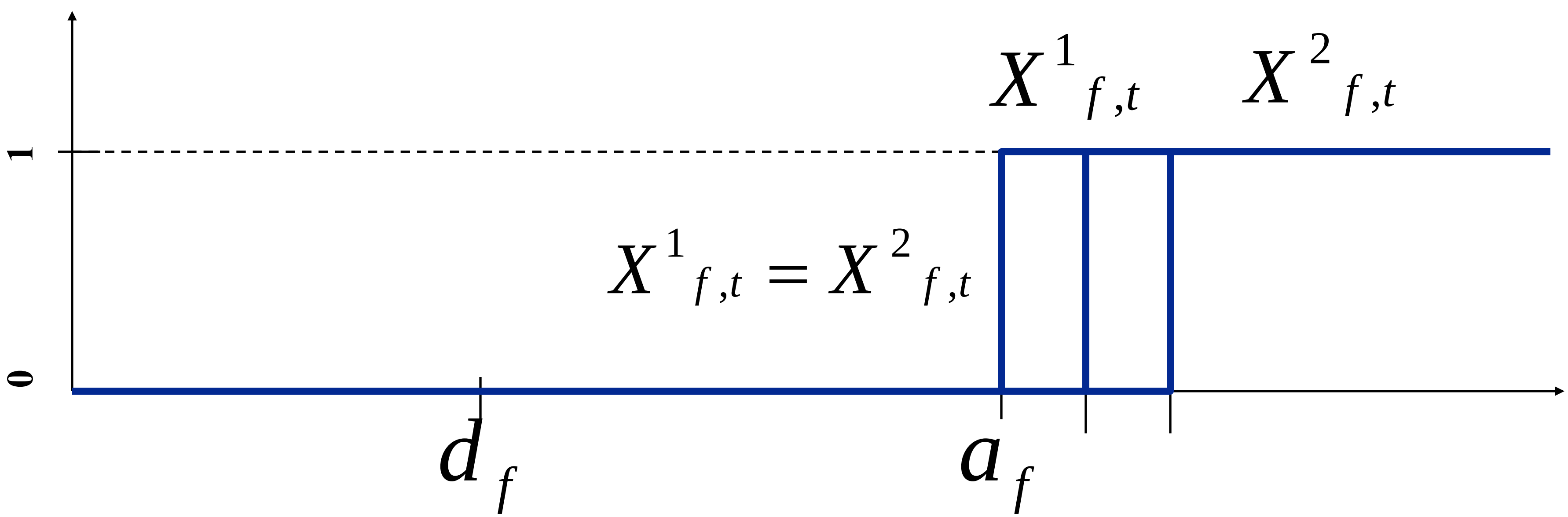


Decision Making Process



Decision Variables

$$X^{q}_{f,t} = \begin{cases} 1 & \text{if flight f is planned to arrive by time period} \\ & \text{t under scenario q;} \\ 0 & \text{otherwise} \end{cases}$$



Model Formulation

- Objective Function
  - Min. Expected Total Cost of Delay (sum of ground and airborne delays)

- Major Constraints
  - Number of arrivals during any time interval (period) less than airport capacity
  - Coupling Constraints: decisions cannot be based on a particular scenario until it is completely realized

Model Parameters and Input Data

$\{1..T+1\}$: set of time periods of uniform duration, T being the planning horizon

$\Phi = \{1..F\}$: Set of Flights

$Dep_f \in \{1..T\}$: scheduled departure time period of flight f

$Arr_f \in \{1..T\}$: scheduled arrival time period of flight f

λ : Cost ratio between airborne and ground delay

Θ : set of capacity scenarios

P_q : Probability of occurrence of scenario $q \in \Theta$

M_t^q : Airport arrival capacity at time period t under capacity scenario q

M_{T+1}^q is set to a high value for all $q \in \Theta$

B = total number of branches of the scenario tree; $B \geq |\Theta|$

N_i = number of scenarios represented by i^{th} branch; $i \in \{1..B\}$

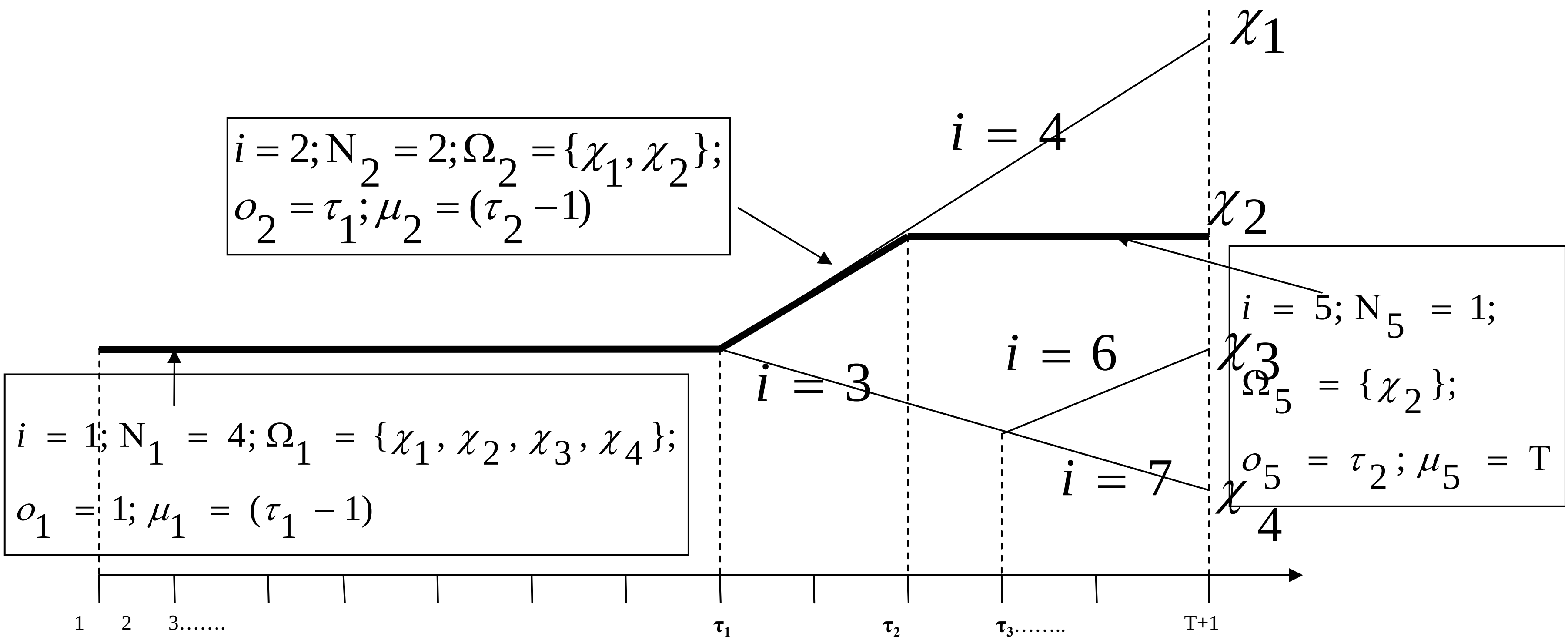
The scenarios represented by branch i is given by set

$$\Omega_i = \{S_1^i, \ldots, S_k^i, \ldots, S_{N_i}^i\}, S_k^i \in \Theta$$

The time periods corresponding to start and end nodes of a branch are given by

o_i *and* $\mu_i; i \in \{1..B\}$

$$B = 7; \Theta = \{\chi_1, \chi_2, \chi_3, \chi_4\}; |\Theta| = 4$$



Decision Variables

$$X_{f,t}^{q} = \begin{cases} 1 & \text{if flight f is planned to arrive by the end of} \\ & \text{time period t under scenario q;} \\ 0 & \text{otherwise} \end{cases}$$

$$q \in \Theta,\ f \in \Phi,$$
$$t \in \{Arr_f..T+1\}$$

$$Y_{f,t}^{q} = \begin{cases} 1 & \text{if flight f is released for departure by the end of} \\ & \text{time period t under scenario q;} \\ 0 & \text{otherwise} \end{cases}$$

$$q \in \Theta,\ f \in \Phi,$$
$$t \in \{Dep_f..T+1\}$$

W_t^q = number of aircraft subject to airborne queuing delay at time t for one or more time periods, under scenario q

Objective Function

$$Min \sum_{q \in \{1..Q\}} P_q \times \left\{ \left[\sum_{f \in \{1..F\}} \sum_{t = Arr_f}^{T+1} \left(t - Arr_f\right) \times \left(X_{f,t}^{q} - X_{f,t-1}^{q}\right) \right] + \lambda \times \sum_{t=1}^{T} W_t^q \right\}$$

Constraints

Decision Variables Non Decreasing

$$X_{f,t}^{q} - X_{f,t-1}^{q} \geq 0; \quad \forall f \in \Phi, q \in \Theta, t \in \{Arr_f..T+1\}$$

Planned Departure Time of Flights

$$Y_{f,t}^{q} = \begin{cases} X_{f,t+Arr_f-Dep_f}^{q}; \; if \quad t + Arr_f - Dep_f \leq T \\ 1 \quad otherwise \end{cases}$$

Arrival Capacity

$$W^{q}_{t-1} - W^{q}_{t} + \sum_{f \in \Phi} \left(X^{q}_{f,t} - X^{q}_{f,t-1} \right) \leq M^{q}_{t}; \quad t \in \{1..T+1\}, q \in \Theta$$

Feasibility Conditions

$$W^{q}_{0} = W^{q}_{T+1} = 0$$

$$X^{q}_{f,T+1} = 1 \qquad \forall f \in \Phi, q \in \Theta$$

Coupling Constraints for Ground Holding Decision Variables

$$Y^{s^{i}_{1}}_{f,t} = ... = Y^{s^{i}_{k}}_{f,t} = ... = Y^{s^{i}_{N_i}}_{f,t}; \qquad f \in \Phi, t \in \{1..T\}; s^{i}_{k} \in \Omega_i : \mathrm{N}_i \geq 2 \text{ and } o_i \leq t \leq \mu_i$$

Static vs Dynamic Formulation

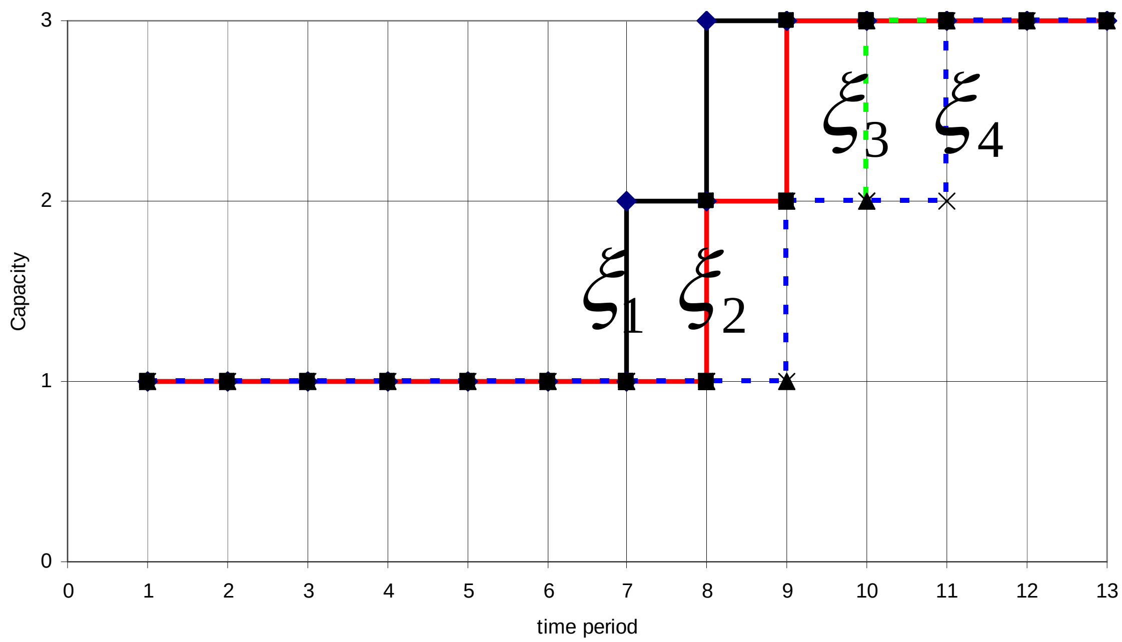
- Static Stochastic Model (Ball et al 2003, Richetta-Odoni 1993) is a special case of dynamic model.
- The decisions are taken once at the beginning of day and not revised later.

$X^{q}_{f,t}$ is same for all $q \in \Theta$; i.e., superscript q in the decision variables can be dropped.

Therefore, $X^{q}_{f,t}$ can be denoted as $X_{f,t}; \forall q \in \Theta$

Similarly, $Y^{q}_{f,t} = Y_{f,t}; \forall q \in \Theta$

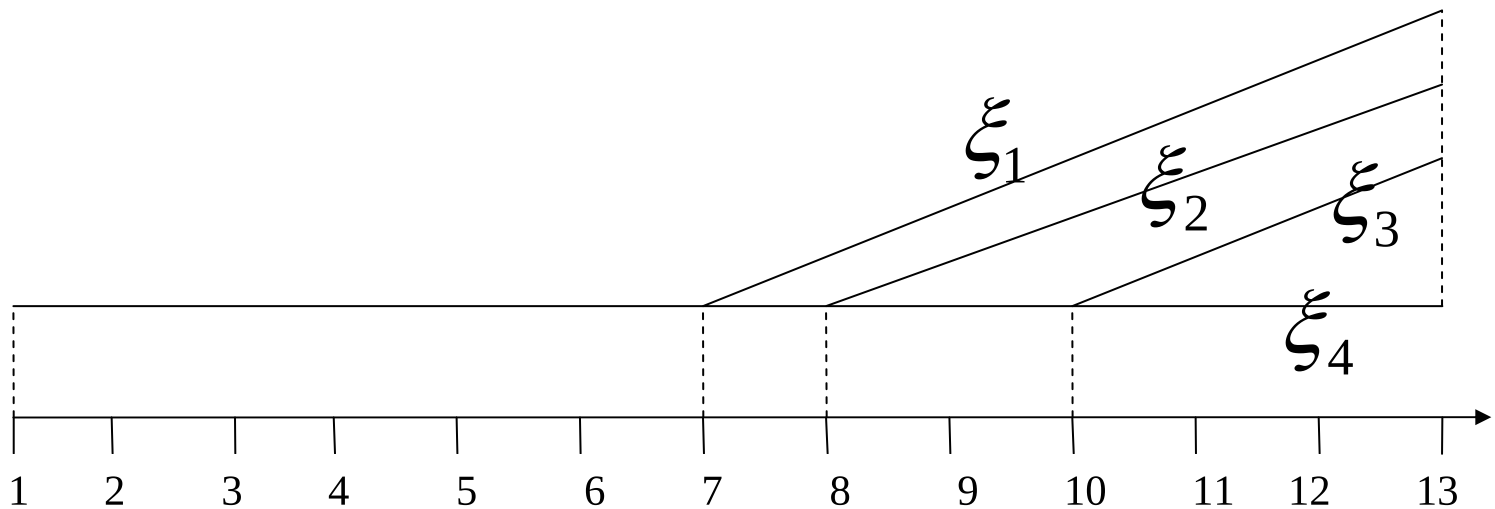
Example



- 4 Scenarios, 4 Decision Stages
- 13 Time Periods
- 13 Flights
- Cost Ratio $\lambda = 5$

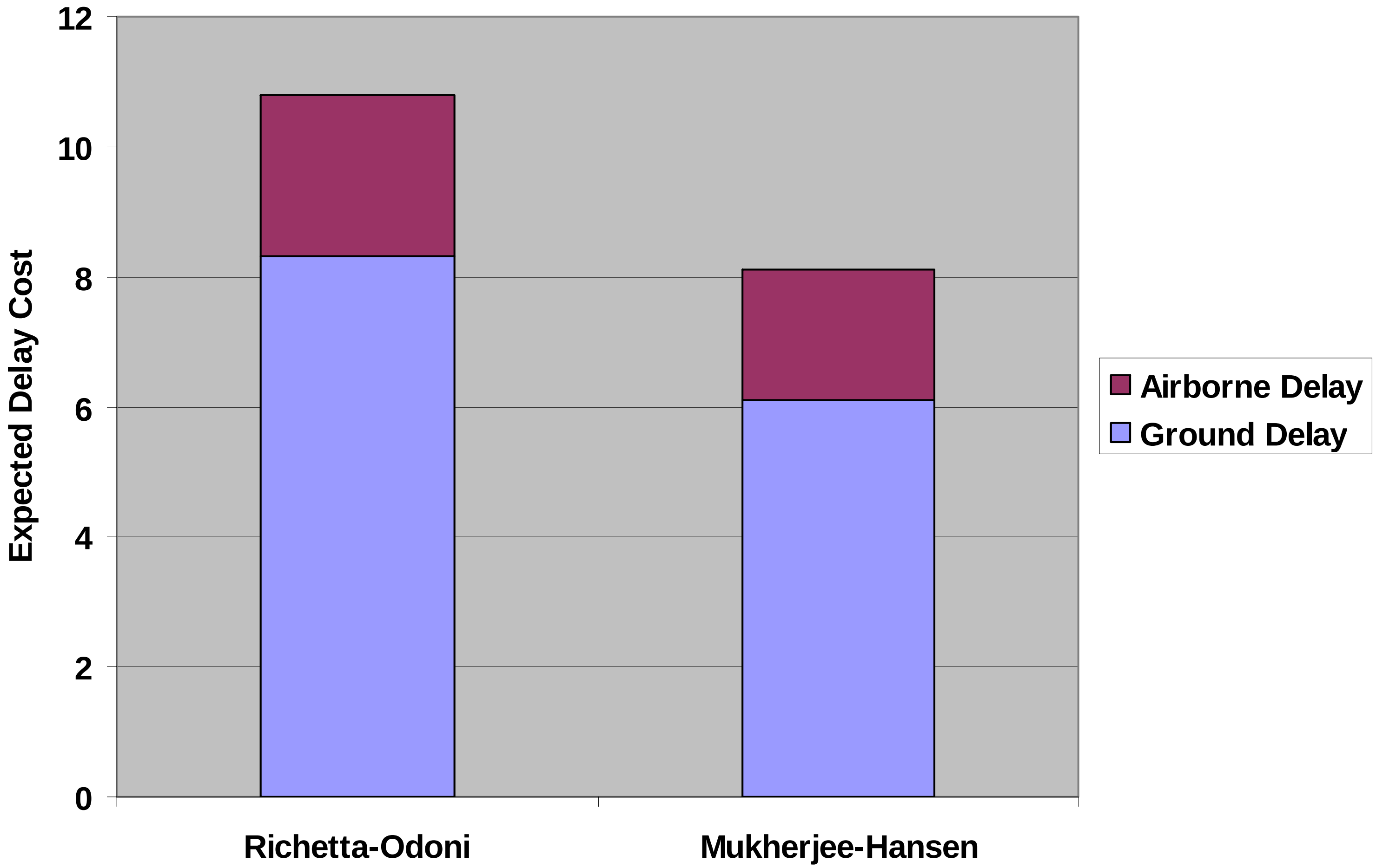
$$\text{Probability Mass Function}: \; P\{\xi_1\} = 0.5; P\{\xi_2\} = 0.3; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1$$

Scenario Tree

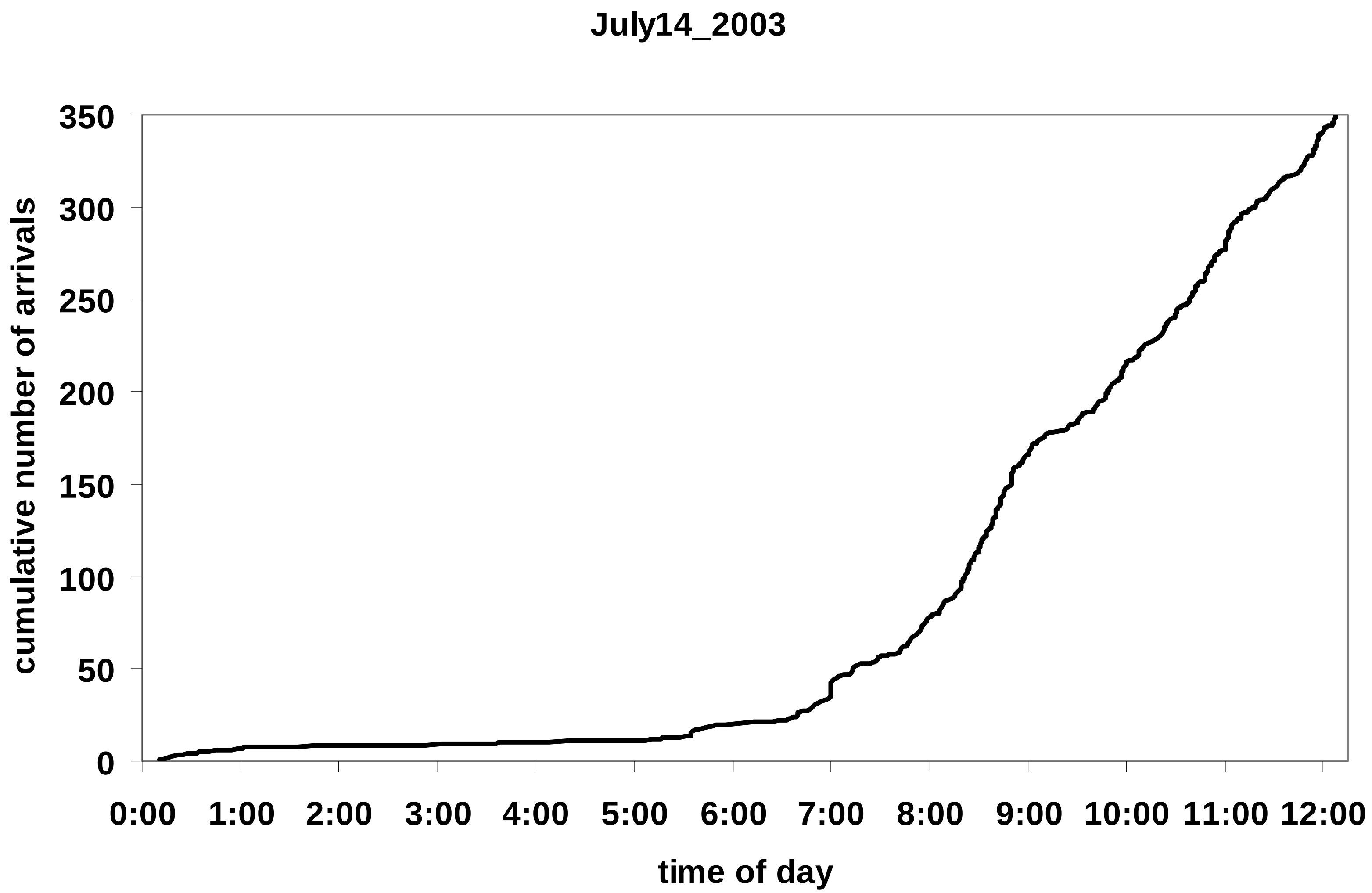


Flight No.	Dep.	Arr.	Decision Stage	**Richetta-Odoni Model**				**Mukherjee-Hansen Model (Optimal Solution 1)**				**Mukherjee-Hansen Model (Optimal Solution 2)**			
				ξ_1	ξ_2	ξ_3	ξ_4	ξ_1	ξ_2	ξ_3	ξ_4	ξ_1	ξ_2	ξ_3	ξ_4
2	***6***	***7***	***1***	***3***	***3***	***3***	***3***	***1***	***2***	***5***	***5***	***1***	***2***	***4***	***4***
3	2	8	1	0	0	0	0	1	1	1	1	1	1	1	1
7	5	9	1	0	0	0	0	0	0	0	0	0	0	0	0
8	***7***	***9***	***2***	***0***	***3***	***3***	***3***	***0***	***1***	***2***	***2***	***0***	***1***	***3***	***3***
12	9	11	3	0	0	1	1	0	0	1	1	0	0	1	1

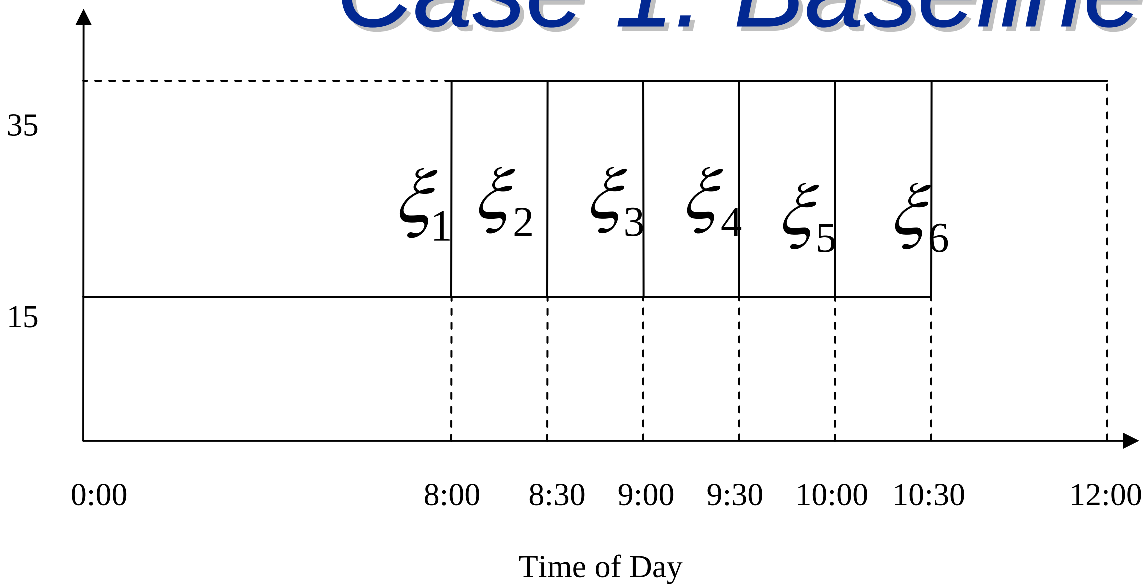
Expected Cost of Delay



Cumulative Arrivals at DFW



Case 1: Baseline

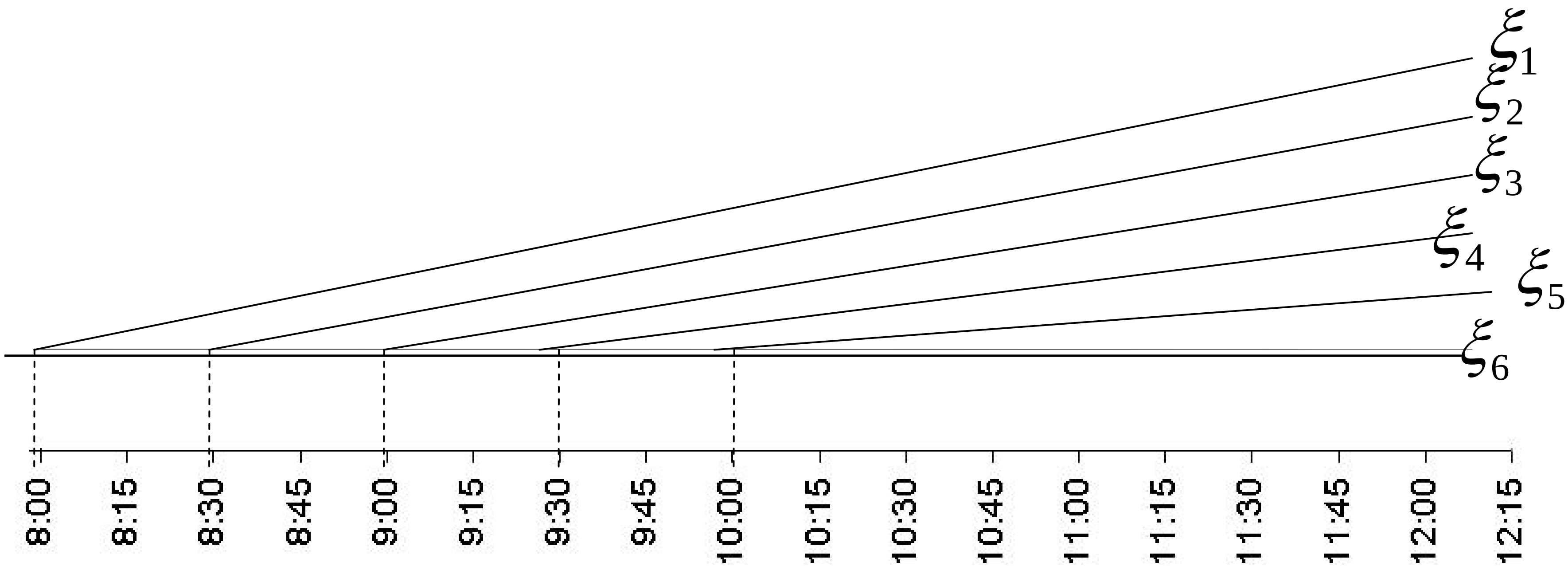


Probability Mass Function

$$P\{\xi_1\} = 0.4; P\{\xi_2\} = 0.2; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1; P\{\xi_5\} = 0.1; P\{\xi_6\} = 0.1$$

Cost Ratio $\lambda = 3$

Scenario Tree for Baseline Case



Other Cases

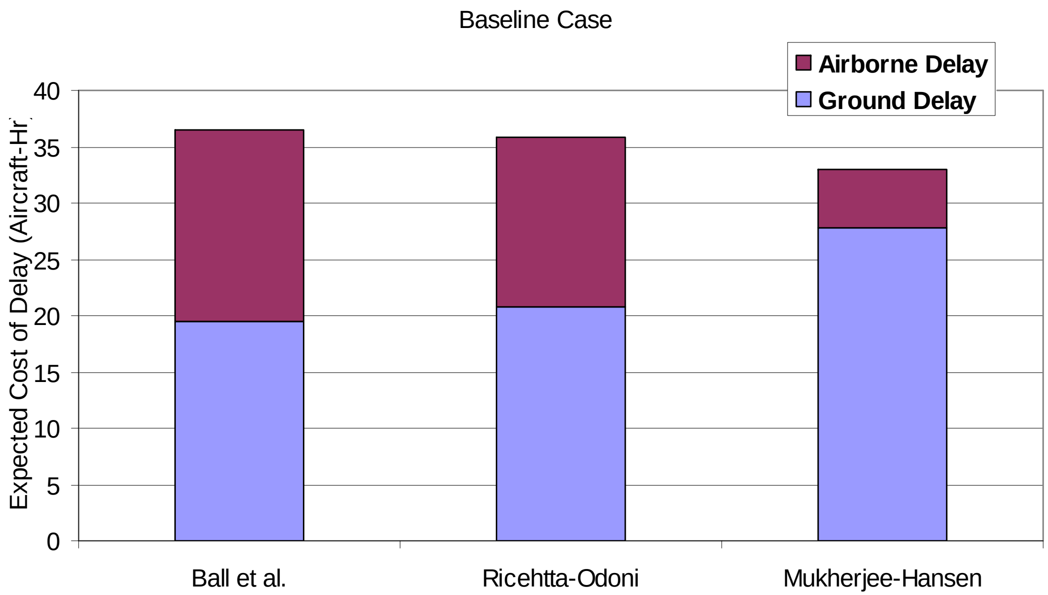
Case 2: Change in Cost Ratio. $\lambda = 25$

Case 3: Change in PMF

$$P\{\xi_1\} = 0.1; P\{\xi_2\} = 0.1; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1; P\{\xi_5\} = 0.2; P\{\xi_6\} = 0.4$$

Case 4: Early Branching. Scenarios are realized 30 minutes earlier

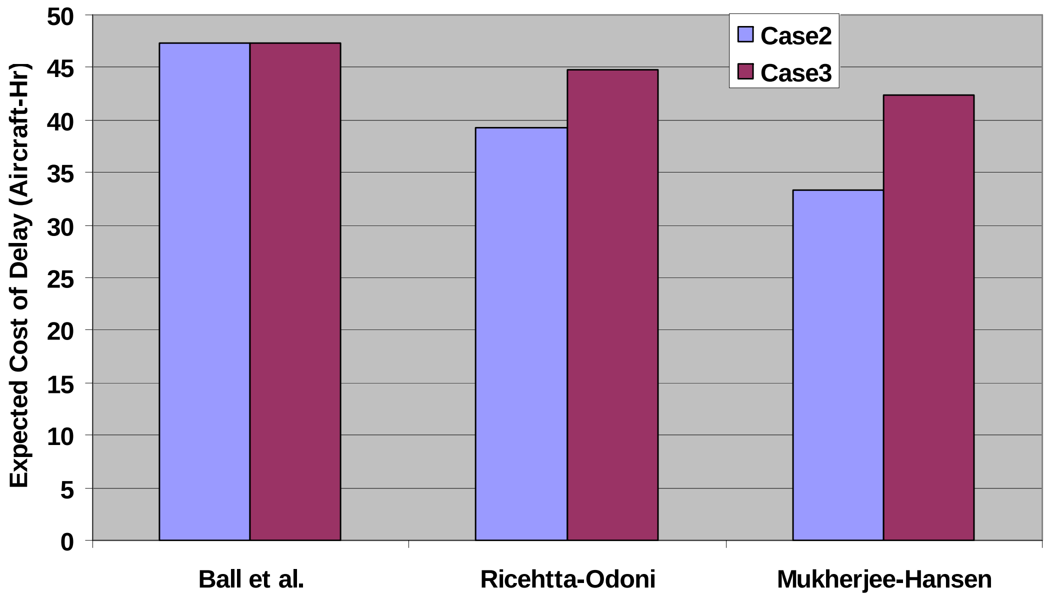
Results: Baseline Case



- Mukherjee-Hansen Model
  - Ground delays more severe
  - Less airborne delays
  - Total expected cost least
- Delay reduction compared to Static Model
  - 10% in Mukherjee-Hansen Model
  - 2% in Richetta-Odoni

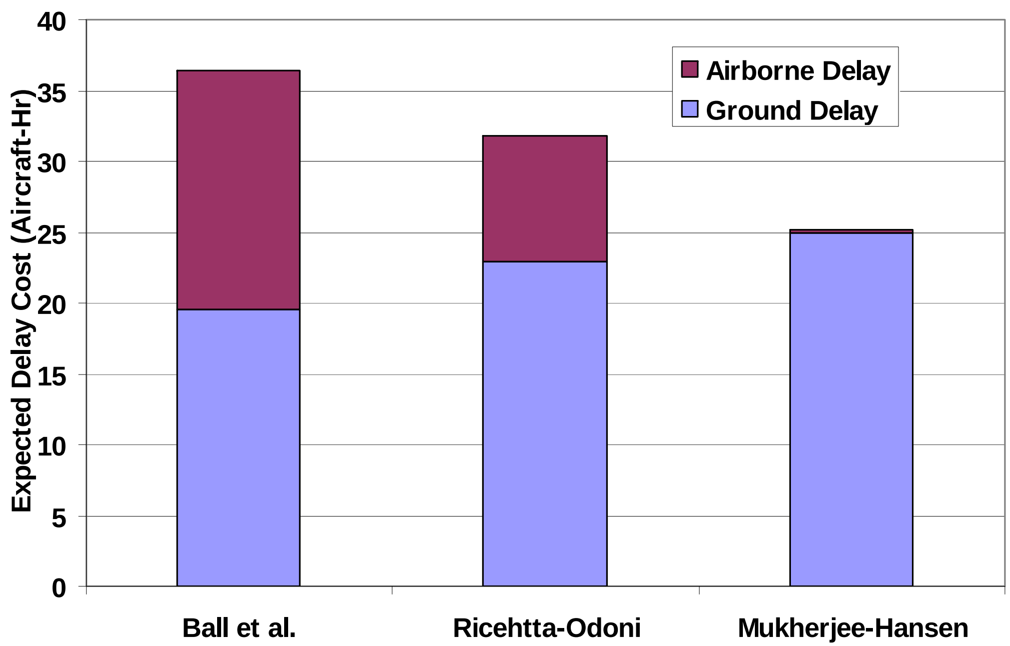
Planned Arrival Rates in Baseline Case												
Time Period	Mukherjee-Hansen Model						Richetta-Odoni Model					
	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
9:00AM-9:15AM	**33**	**25**	**25**	**25**	**25**	**25**	**35**	**34**	**34**	**34**	**34**	**34**
9:15 AM-9:30 AM	16	5	5	5	5	5	14	14	14	14	14	14
9:30AM-9:45AM	12	26	18	18	18	18	12	12	12	12	12	12
9:45AM-10:00AM	20	25	12	12	12	12	20	21	13	13	13	13
10:00AM-10:15AM	**12**	**12**	**33**	**33**	**33**	**33**	**12**	**12**	**20**	**20**	**20**	**20**

Cases 2 and 3



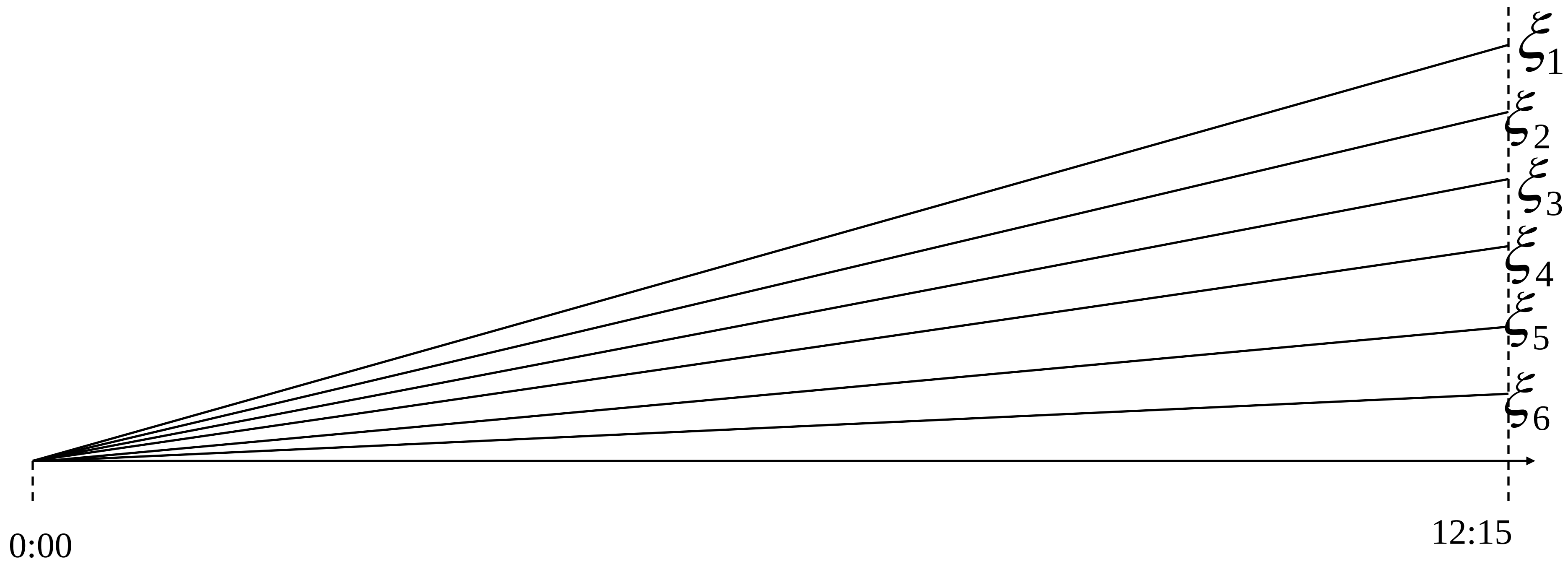
- No airborne delays
- Static model plans for worst scenario
- Dynamic models adaptive to changing conditions
- Delays are higher in case 3 due to high probability of worse conditions

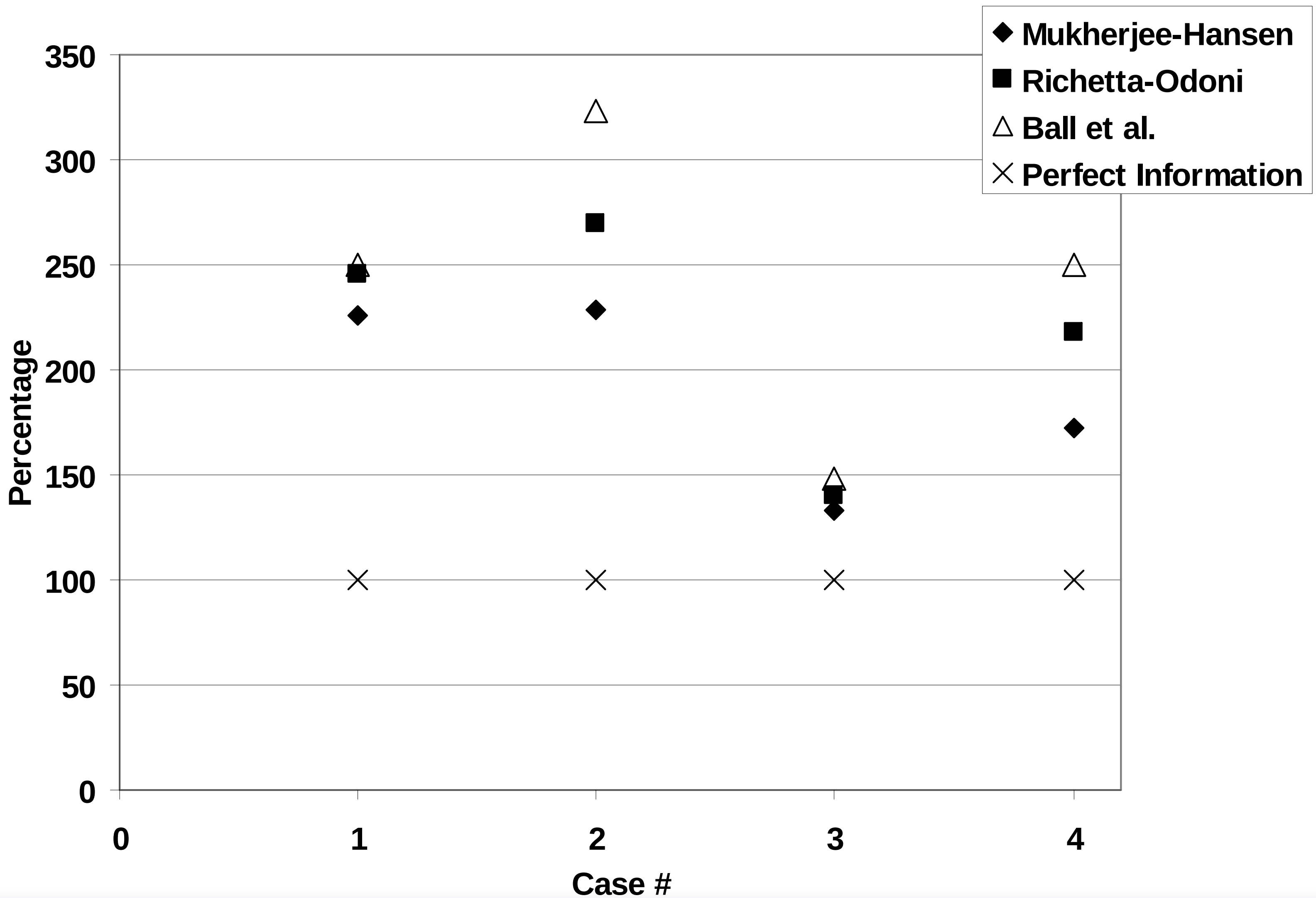
Case 4: 30 Minutes Early Information



- Static model produces same delays as in baseline case
- Value of early information
- Mukherjee-Hansen Model
  - Least delays
  - Absorbs most of the delay by ground holding

Perfect Information Case





350
300
250
200
150
100
50
0
Percentage
0
1
2
3
4
Case #
Mukherjee-Hansen
Richetta-Odoni
Ball et al.
Perfect Information

Alternative Objective Functions

Minimizing Expected Squared Deviation from RBS Allocation

$$\min_{q \in \{1..Q\}} \sum P\{q\} \times \left\{ \left[\sum_{f \in \{1..F\}} \sum_{t = Arr_f}^{T+1} \left(t - RBS_f^q \right)^2 \times (X_{f,t}^q - X_{f,t-1}^q) \right] + \lambda \times \sum_{t=1}^{T} W_t^q \right\}$$

Multi-Criteria Optimization

$$\min_{q \in \{1..Q\}} \sum P\{q\} \times \left\{ \begin{array}{l} \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - Arr_f\right) \times \left(X_{f,t}^{q} - X_{f,t-1}^{q}\right) \right] + \\ weight \times \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - RBS_f^{q}\right)^2 \times \left(X_{f,t}^{q} - X_{f,t-1}^{q}\right) \right] + \\ \lambda \times \sum_{t=1}^{T} W_t^{q} \end{array} \right\}$$

Work in Progress

- Reformulating the model as a minimum cost network flow problem.

- Ability to handle time varying unconditional probabilities of the capacity scenarios

Acknowledgements

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