Control Theoretic Tool for Trajectory Reconstruction

BISWADIP DEY (biswadip@umd.edu)

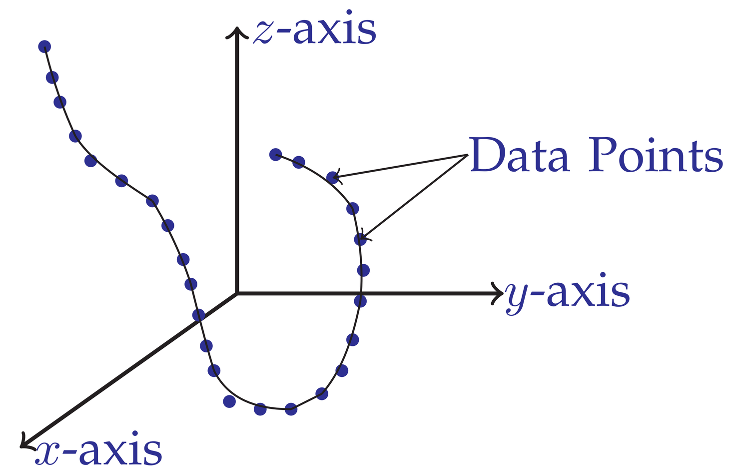
- A joint work with Prof. P. S. Krishnaprasad.

PROBLEM

Given a time series of observed positions $\{r_i\}_{i=0}^N$ in three dimensional space, our primary objective is to generate a smooth trajectory to fit these data points.

- A penalty term is introduced to assure smoothness of the reconstructed trajectory.

$$\underbrace{\sum_{i=0}^{N} \|r(t_i) - r_i\|^2}_{\text{Fit Error}} + \lambda \underbrace{\int_{t_0}^{t_N} (\text{Suitable Path Cost})\, dt}_{\text{Penalty Term}}$$



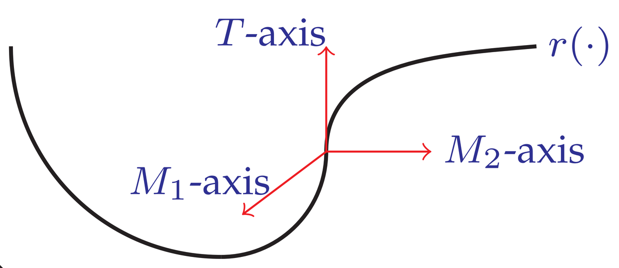
- **BUT** the relative importance of the fit error with respect to the penalty term is not known *a priori*.
- The optimal value of the regularization parameter (λ) is chosen using ordinary cross validation, and the optimal value depends on the signal-to-noise ratio in the data.

GENERATIVE MODEL I

$$\begin{aligned} \dot{r} &= \nu T \\ \dot{T} &= \nu (k_1 M_1 + k_2 M_2) \\ \dot{M}_1 &= -\nu k_1 T \\ \dot{M}_2 &= -\nu k_2 T \end{aligned} \qquad (1)$$

- **Penalty Term** to ensure smoothness:

$$\int_{t_0}^{t_N} \left(\dot{k}_1^2 + \dot{k}_2^2 + \dot{\nu}^2\right) dt$$



GENERATIVE MODEL II

$$\begin{aligned} \dot{r} &= v \\ \dot{v} &= a \\ \dot{a} &= u \end{aligned} \qquad (2)$$

- **Penalty Term** to ensure smoothness:

$$\int_{t_0}^{t_N} (u^T u)\, dt$$

MODEL I → MODEL II

$$\begin{aligned} v &= \nu T \\ a &= \dot{\nu} T + \nu^2 k_1 M_1 + \nu^2 k_2 M_2 \\ u &= (\ddot{\nu} - \nu^3 (k_1^2 + k_2^2)) T \\ &\quad + (3\nu\dot{\nu} k_1 + \nu^2 \dot{k}_1) M_1 \\ &\quad + (3\nu\dot{\nu} k_2 + \nu^2 \dot{k}_2) M_2 \end{aligned}$$

MODEL II → MODEL I

$$\nu = \|v\|$$
$$T = \frac{v}{\|v\|}$$
$$\dot{T} = \frac{1}{\nu}(a - (a \cdot T)T)$$
$$\kappa = \frac{\|\dot{T}\|}{\nu}$$
$$\tau = \frac{v \cdot (a \times u)}{\|v \times a\|^2}$$

RECONSTRUCTION THROUGH ERROR MINIMIZATION

Model I (Nonlinear)

- Approximation by **piecewise constant** speed and curvature, transforms the problem into a **non-convex numerical optimization problem** [2].
- MATLAB routine: ***fminunc***.
- The algorithm is capable of estimating curvature with higher resolution, but the process is **time consuming**.

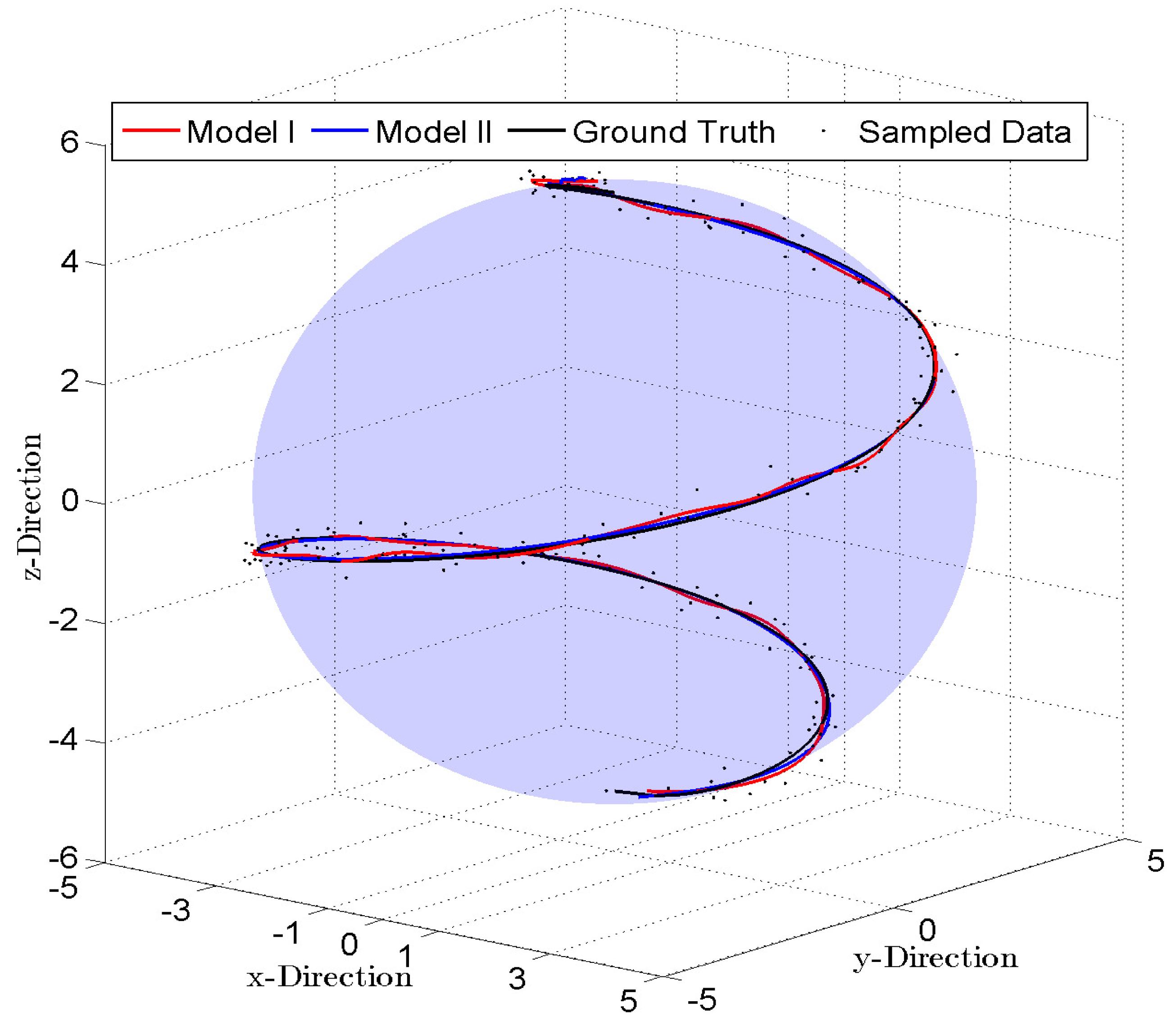
Model II (Linear)

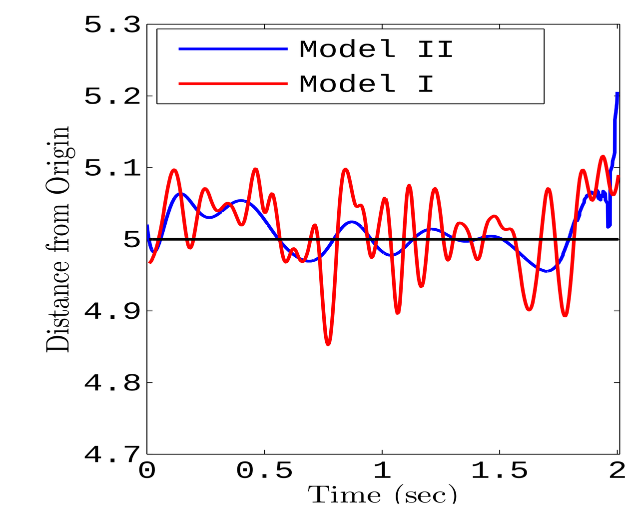
- *Path-independence lemmas* and *Riccatti equation* **ensure global optimality of the solution** and **the solution is semi-analytic** [1].
- Reconstructed positions can be expressed as **linear combinations of raw data**, but the **linear weights vary across data points**.
- This method is **orders of magnitude faster** than the nonlinear version of the story.

CHOOSING A REGULARIZATION PARAMETER (OCV)

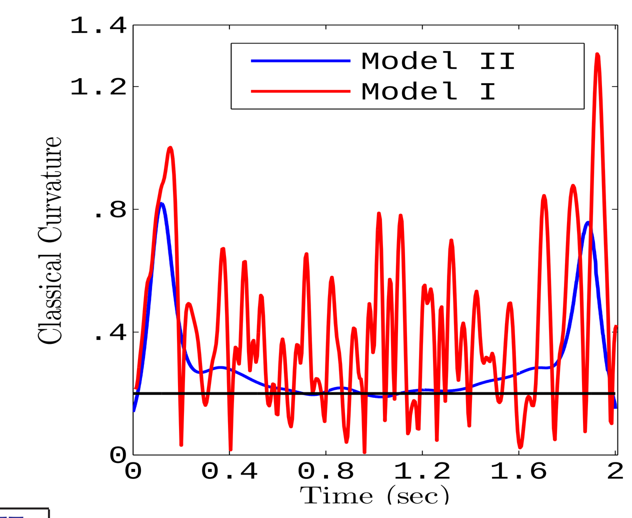
- We use a subset of the given dataset to obtain a parameter estimate and use the rest of the data for performance validation under that particular value of the estimate.
- Here we use leaving-out-one strategy.
- Each data point is left out in turn and an estimate for the curve is derived from the rest of the data. The prediction error is computed at the left out data point and they are summed to yield the ordinary cross validation cost. An optimal λ minimizes the total prediction error (sampled variance).

RESULTS (SYNTHETIC DATA - CURVE ON A SPHERE)





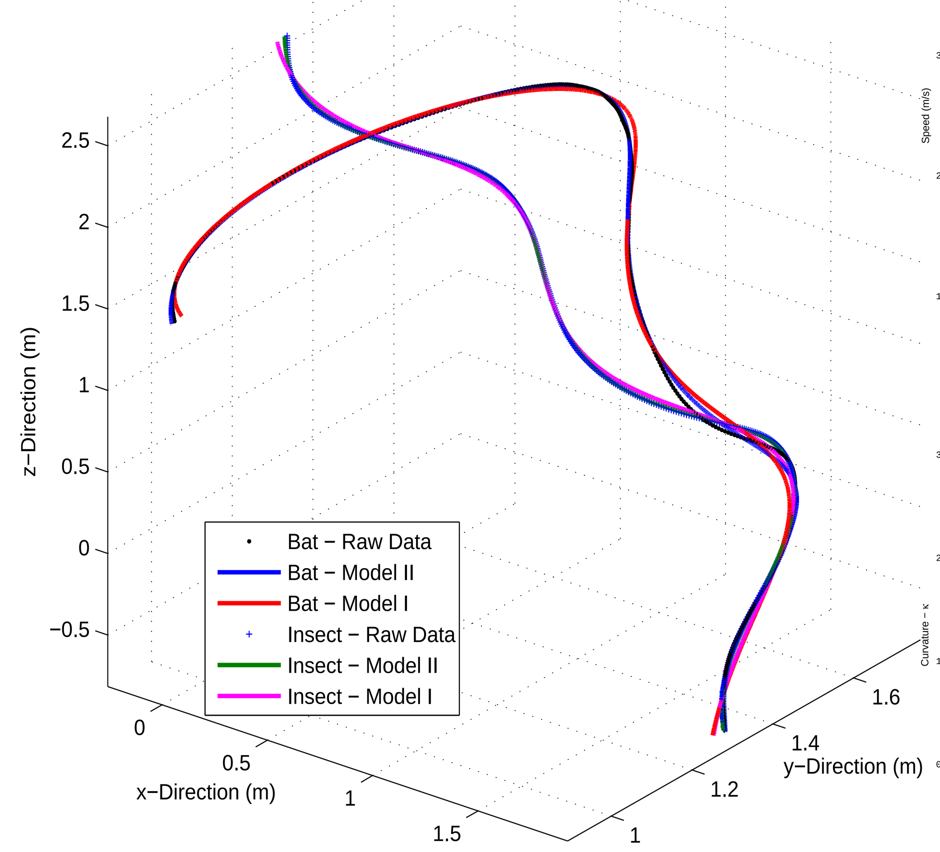
Distance from Origin



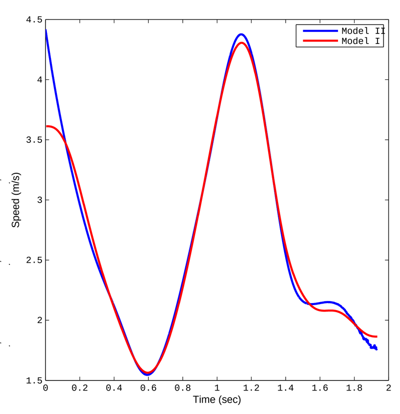
Classical Curvature (κ)

	Model I	Model II
Avg. Fit Error	0.0599	0.0684

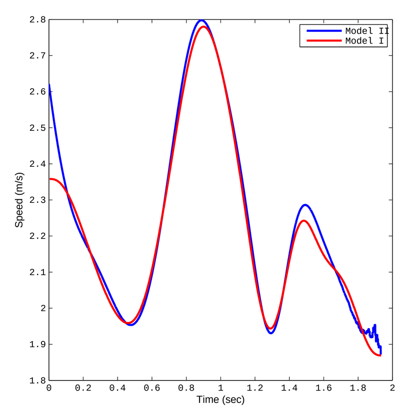
RESULTS (BAT-MANTIS PURSUIT EVENT)



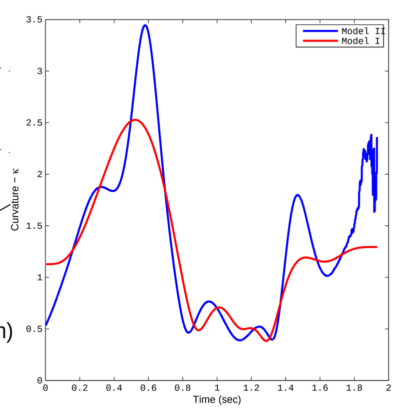
ng



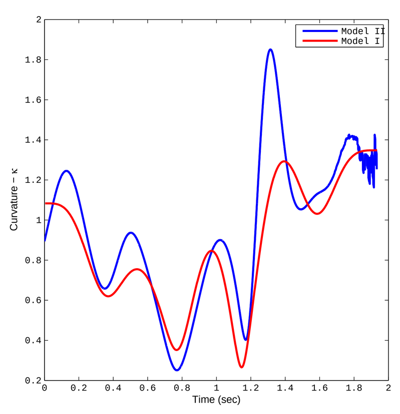
Speed (Bat)



Speed (Insect)



Curvature (Bat)



Curvature (Insect)

Avg. Fit Error	Model I	Model II
Bat	2.2401×10^{-4}	7.6142×10^{-5}
Mantis	1.2966×10^{-4}	2.3913×10^{-5}

WORK IN PROGRESS

- We are working with generative model I to reconstruct trajectories in a semi-analytic way.

SOURCE CODE

The source code and compiled executables with an interactive interface are available at `http://www.isr.umd.edu/Labs/ISL/SMOOTHING/`

ACKNOWLEDGEMENT

This research was supported in part by the Air Force Office of Scientific Research under AFOSR Grant No. FA95501010250.

REFERENCES

[1] B. Dey, P. S. Krishnaprasad. *Trajectory Smoothing as a Linear Optimal Control Problem.* in Proceedings of *Allerton Conference on Communication, Control, and Computing*, 2012.

[2] P.V. Reddy. *Steering Laws for Pursuit.* Masters' Thesis, University of Maryland, 2007.