Sensor Network Platform Positioning with Cyclic Pursuit

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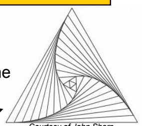
Abstract

Pursuit is often viewed as a competitive phenomenon, whether it is observed in the biological setting or in the context of unmanned vehicle maneuvers or weapons engagements. Here, we demonstrate that pursuit can actually serve as a building block for cohesion, generating complex group behavior on a larger scale through local interactions of individual agents. The resultant group behavior could serve as the basis for positioning ***a formation of vehicles carrying elements of a mobile sensor network***. In the particular case, we investigate an *n*-agent ***cyclic pursuit*** scheme (i.e. agent *i* pursues agent *i+1*, modulo *n*) in which a ***constant bearing angle pursuit strategy*** (originally developed by Wei, Justh, and Krishnaprasad, 2008) is employed by agents modeled as unit mass particles traveling at constant speed in the plane. We demonstrate the existence of a 2*n*-dimensional invariant submanifold within the state space and derive necessary and sufficient conditions for the existence of rectilinear and circling relative equilibria on that manifold.

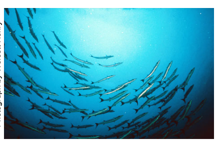
Cyclic Pursuit

In an *n*-agent cyclic pursuit scheme, each agent (e.g. unmanned vehicle, fish, etc.) pursues the next agent in the group, with agent *n* pursuing agent 1.



Courtesy of John Sharp

Example: For three agents using exact classical pursuit (i.e. velocity is directed towards the pursuee's current position), it can be proved that they will meet simultaneously at a Brocard point of the initial triangular configuration.

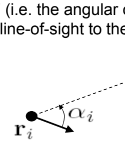


Photograph by Wolcott Henry

It's possible that certain group behaviors in biology (e.g. circling movement of a school of fish) are based on a cyclic pursuit scheme.

Constant Bearing (CB) Pursuit

In our scheme, the agents use a *constant bearing (CB) pursuit strategy*, maneuvering to maintain a specified "target bearing" (i.e. the angular offset between their velocity vector and the line-of-sight to the pursuee.)



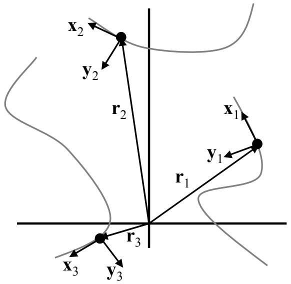
$\alpha_i \in [0, 2\pi] =$ desired CB pursuit angle

System Model

$$\begin{aligned}\dot{\mathbf{r}}_i &= \mathbf{x}_i,\\ \dot{\mathbf{x}}_i &= \mathbf{y}_i u_i, \qquad (1)\\ \dot{\mathbf{y}}_i &= -\mathbf{x}_i u_i, \quad i = 1, 2, \ldots, n\end{aligned}$$

$\mathbf{r}_i$ = position of i^{th} particle
$\mathbf{x}_i$ = unit tangent vector
$\mathbf{y}_i$ = unit normal vector
u_i = steering (curvature) control

$$\mathbf{r}_i \neq \mathbf{r}_{i+1}$$



CB Control Law

Introduce a cost function to indicate "distance" from desired CB pursuit state:

$$\Lambda_i = \underbrace{R(\alpha_i)}\mathbf{x}_i \cdot \underbrace{\frac{\mathbf{r}_{i,i+1}}{|\mathbf{r}_{i,i+1}|}} \qquad (2)$$

$\Lambda_i = -1$ ⇕ Attainment of CB pursuit state

Rotation matrix: rotates vector $\mathbf{x}_i$ *by* α_i *radians in the CCW direction*

$\mathbf{r}_{i,j} \triangleq \mathbf{r}_i - \mathbf{r}_j$

Control Law for CB Pursuit

$$u_i = -\mu_i \left(R(\alpha_i)\mathbf{y}_i \cdot \frac{\mathbf{r}_{i,i+1}}{|\mathbf{r}_{i,i+1}|} \right) - \frac{1}{|\mathbf{r}_{i,i+1}|} \left(\frac{\mathbf{r}_{i,i+1}}{|\mathbf{r}_{i,i+1}|} \cdot \dot{\mathbf{r}}_{i,i+1}^{\perp} \right) \qquad (3)$$

control gain

"perp" notation indicates CCW rotation by $\pi/2$

Under the CB control law, the cost function evolves according to

$$\dot{\Lambda}_i = -\mu_i \left(1 - \Lambda_i^2\right) \qquad (4)$$

and the ***CB⁻ submanifold*** (defined below) is invariant.

$$CB^- = \{(\mathbf{r}_1, \mathbf{x}_1, \mathbf{y}_1, \ldots, \mathbf{r}_n, \mathbf{x}_n, \mathbf{y}_n) \mid \Lambda_i = -1, i = 1, 2, \ldots, n\}$$

Reduction by the Symmetry Group SE(2)

Since our dynamics (1) and control law (3) are SE(2)-invariant, we can define a new set of "shape variables" that depend only on relative state:

$$\begin{aligned}\phi_i &= \mathbf{x}_i \cdot \mathbf{x}_{i+1},\\ \gamma_i &= \mathbf{x}_i \cdot \mathbf{y}_{i+1},\\ \beta_i &= \mathbf{x}_i \cdot \frac{\mathbf{r}_{i,i+1}}{|\mathbf{r}_{i,i+1}|}, \qquad (5)\\ \delta_i &= \mathbf{y}_i \cdot \frac{\mathbf{r}_{i,i+1}}{|\mathbf{r}_{i,i+1}|},\\ \rho_i &= |\mathbf{r}_{i,i+1}|, \quad i = 1, 2, \ldots, n\end{aligned}$$

Additional relationships:
$$\phi_i^2 + \gamma_i^2 = 1$$
$$\beta_i^2 + \delta_i^2 = 1$$

Reduced Shape Dynamics on CB^-

On the invariant CB^- submanifold, we have

$$\beta_i \equiv -\cos(\alpha_i), \quad \delta_i \equiv -\sin(\alpha_i)$$

which yields a set of reduced "shape" dynamics given by

$$\begin{aligned}\dot{\phi}_i &= -\gamma_i \Big[\frac{1}{\rho_i} \Big((1-\phi_i)\sin(\alpha_i) - \gamma_i \cos(\alpha_i) \Big) - \\ &\qquad \frac{1}{\rho_{i+1}} \Big((1-\phi_{i+1})\sin(\alpha_{i+1}) - \gamma_{i+1}\cos(\alpha_{i+1}) \Big) \Big],\\ \dot{\gamma}_i &= \phi_i \Big[\frac{1}{\rho_i} \Big((1-\phi_i)\sin(\alpha_i) - \gamma_i \cos(\alpha_i) \Big) - \qquad (6)\\ &\qquad \frac{1}{\rho_{i+1}} \Big((1-\phi_{i+1})\sin(\alpha_{i+1}) - \gamma_{i+1}\cos(\alpha_{i+1}) \Big) \Big],\\ \dot{\rho}_i &= -(1-\phi_i)\cos(\alpha_i) - \gamma_i \sin(\alpha_i), \quad i = 1, 2, \ldots, n.\end{aligned}$$

Initial analysis and simulations suggest that the CB⁻ submanifold is attractive on a large region of the state space, and therefore we focus our analysis on these reduced dynamics.

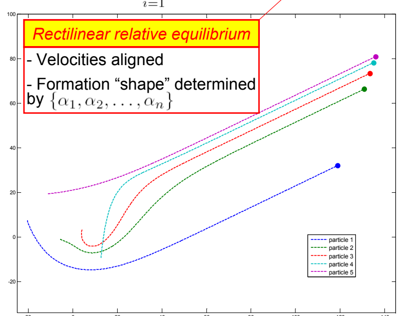
Relative Equilibria

Equilibria of the shape dynamics
=
Relative equilibria for the full system dynamics

Proposition 1.1. *Given* $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, *a relative equilibrium corresponding to rectilinear motion on* CB^- *exists for system (1) under* $CB(\alpha)$ *control law (3) if and only if there exists a set of constants* $\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ *such that* $\sigma_i > 0$, $i = 1, 2, \ldots, n$ *and*

$$\sum_{i=1}^{n} \sigma_i e^{j(\alpha_i)} = 0.$$

Rectilinear relative equilibrium
- Velocities aligned
- Formation "shape" determined by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$

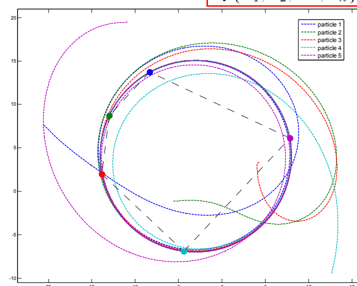


Proposition 1.2. *Given* $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, *a relative equilibrium corresponding to circling motion on a common orbit on* CB^- *exists for system (1) under* $CB(\alpha)$ *control law (3) if and only if*

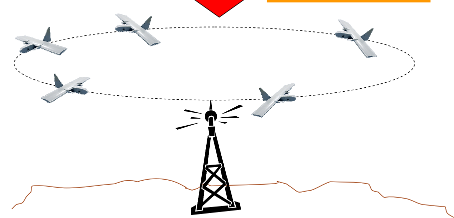
i. $\sin(\alpha_i) > 0 \;\; \forall i \in \{1, 2, \ldots, n\}$ *or*
$\sin(\alpha_i) < 0 \;\; \forall i \in \{1, 2, \ldots, n\}$,

ii. $\sin\left(\sum_{i=1}^{n} \alpha_i\right) = 0.$

Circling relative equilibrium
- Platforms travel on a common circular trajectory
- Formation "shape" determined by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$



Sensor network formation



Ongoing & Future Work

- Investigation of stability/convergence properties of relative equilibria and particular invariant manifolds
- Characterization of system behavior for parameters that do not satisfy the conditions of Proposition 1.1 or 1.2
- Examination of the affects of programming constant bearing angle offsets over time