Stabilization of Networked Control Systems: Communication and Controller co-design

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(Joint work with Lei Zhang)

Research Directions

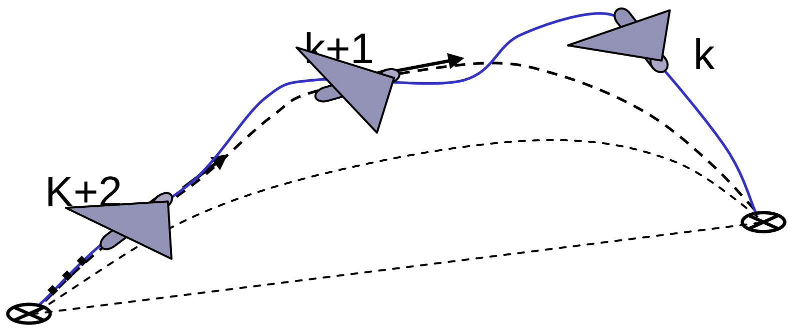
- Interaction of Communication and Control (with Lei Zhang)
  - Exploring control and communication as a joint problem.
  - How to design communication and control laws while avoiding the computational complexity that can arise.
  - "Static" vs. "feedback-based" communication patterns for control.

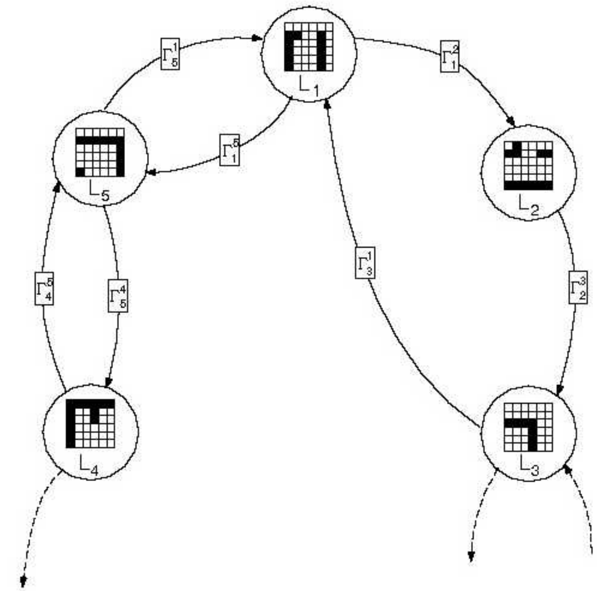
- *From "individual" to "group"-level control* (with Cheng Shao)
  - Solving optimal control problems by neighbor-to-neighbor interaction
  - What can groups do that individuals cannot?
  - Free final time, partially-constrained final state problems.

- The problem of *tokenizing robot control* (with Sean Andersson)
  - Control laws must be "composed" to solve problems of practical importance in real-world environments (e.g. robot navigation).
  - Control instructions as a means of representing geographical relationships.
  - Language-based navigation under sensor/actuator/environment uncertainty.

•Optimal control by “local pursuit”

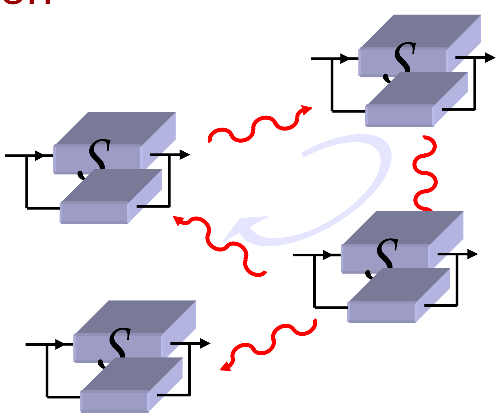
•Exploring the limits of some biologically-inspired control laws





•Control with limited communication

•Tools for co-designing Control and Communication



•Motion Description Languages

•Writing hierarchically-composable, machine-independent control programs for hybrid systems

•Landmark-based navigation / localization

Related Publications (since last year's meeting)

- D. Hristu-Varsakelis, "**Feedback control with Communication Constraints**", to appear in the Handbook of Networked and Embedded Control Systems, D. Hristu-Varsakelis and W. S. Levine, eds, Birkhauser, Boston, 2005.

- L. Zhang and D. Hristu-Varsakelis, "**Stabilization of Networked Control Systems: Communication and Controller co-design**", Subm. to the 2005 American Control Conference

- L. Zhang and D. Hristu-Varsakelis, "**Stabilization of Networked Control Systems under Feedback-based Communication**", Subm. to the 2005 IFAC World Congress.

- D. Hristu-Varsakelis and W. S. Levine, "**A Laboratory Course for Networked Digital Control Systems**", to appear in the IEEE Control Systems Magazine.

- S. Andersson and D. Hristu-Varsakelis, "**Stochastic Language-based Motion Control**", IEEE CDC 2003.

- D. Hristu-Varsakelis, M. Egerstedt, P. S. Krishnaprasad, "**On the Structural Complexity of the Motion Description Language MDLe**", IEEE CDC 2003.

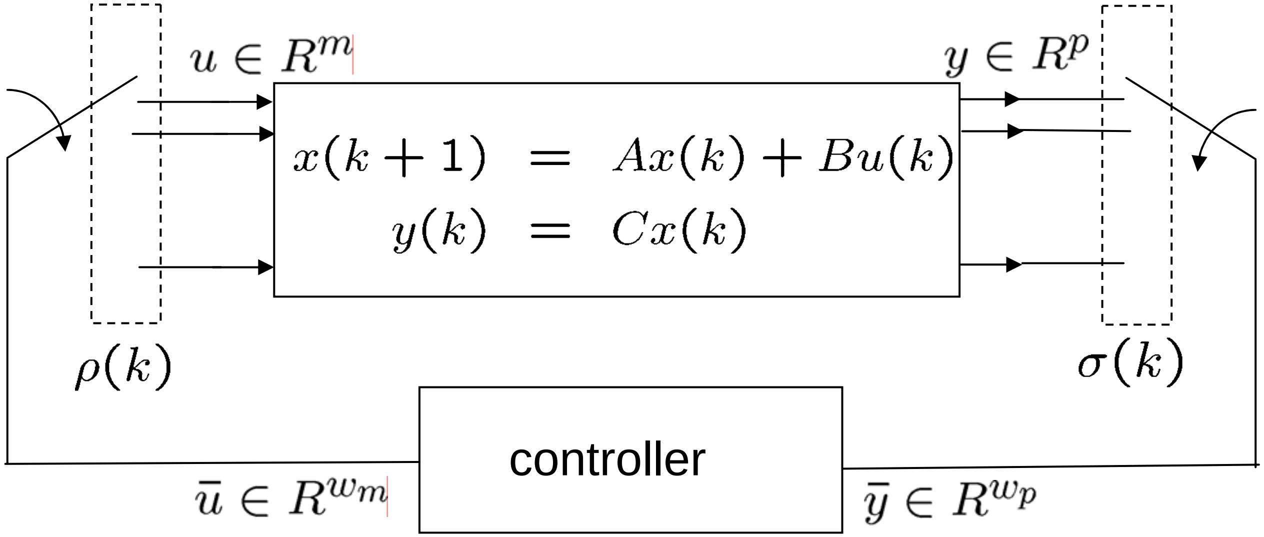
- S. Andersson and D. Hristu-Varsakelis, "**Language-based Feedback Control using Monte Carlo Sensing** ", subm. to IEEE Conf. on Robotics and Automation, 2005.

- D. Hristu-Varsakelis and C. Shao, "**Biologically-inspired optimal control: learning from social insects**" to appear in the International Journal of Control.

- C. Shao and D. Hristu-Varsakelis, "**Biologically-inspired optimal control via intermittent cooperation** ", Subm. to the 2005 American Control Conference.

- C. Shao and D. Hristu-Varsakelis, "**Optimal control through biologically inspired pursuit**", Subm. to the 2005 IFAC World Congress

Stabilization of NCS: Modeling Communication Constraints



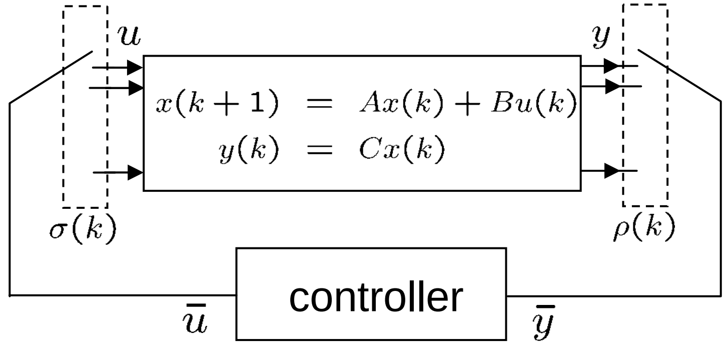
- $u \neq \bar{u}$ and $y \neq \bar{y}$, due to limited communication.
- Define an *M-to-N* *communication sequence*:

  a map, $\sigma(k): Z \mapsto \{0,1\}^M$, satisfying $||\sigma(k)||^2 = N, \ \forall k$.

- Let $\rho(k)$, $\sigma(k)$, be w_m–to-m, and p-to-w_p communication sequences, where $w_m < m$, $w_p < p$.

- Goal: Design a stabilizing controller and pair of communication sequences

Stabilization of NCS: Modeling Communication Constraints



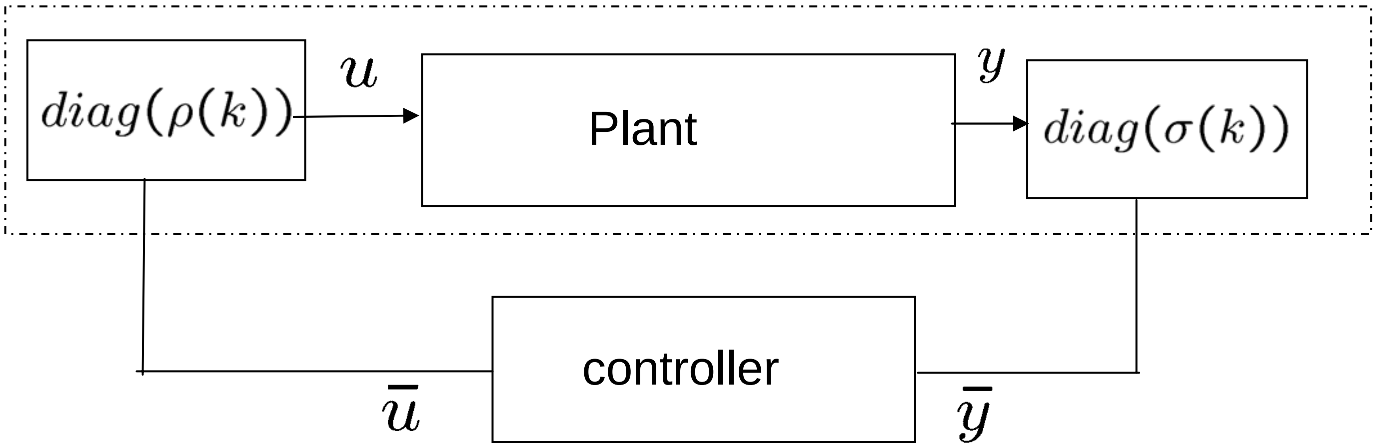
What to do with sensors/actuators that are not communicating?

- Hold (ZOH) signals to their value(s) at the time of the last "interruption".
  - Communication and Control become tightly coupled.
  - Computational complexity of stabilization problem is high
  - Feedback-based communication policies in limited settings (e.g. block-diagonal plant and controller)

    (see works by Brockett, Hristu & Morgansen, Ishii & Francis, Hristu & Kumar).

- Alternative: "Ignore" sensors/actuators that are not actively transmitting.
  - Have the plant (controller) set corresponding actuators (sensor readings) to *zero*.
  - This will reduce the complexity (but requires additional processing at the plant, e.g., recognizing which actuators should be turned off).

Low-complexity Communication/Controller Co-design



- Without ZOH, the effects of limited communication are very much tractable:

$$\bar{y}(k) = diag(\sigma(k)) \cdot y(k) \qquad \bar{u}(k) = diag(\rho(k)) \cdot u(k)$$

- From the point of view of the controller, the system is now LTV:

$$\begin{aligned} x(k+1) &= Ax(k) + B\ diag(\rho(k))\bar{u}(k) \\ \bar{y}(k) &= diag(\sigma(k))Cx(k) \end{aligned}$$

We know how to design stabilizing controllers for LTV (e.g. periodic) linear systems, but how should we choose the communication sequences to ensure/preserve stabilizability?

•Examine the state evolution over k_f steps:

$$x(k_f) = \mathcal{C} \cdot \left[\begin{array}{cccc} \bar{u}^T(0) & \bar{u}^T(1) & \cdots & \bar{u}^T(k_f - 1) \end{array} \right]^T$$ where:

$$\mathcal{C} = [A^{k_f-1}BM_\rho(0),\ A^{k_f-2}BM_\rho(1), \cdots,\ BM_\rho(k_f-1)]$$

and $M_\rho(k) \stackrel{\triangle}{=} diag(\rho(k))$

- The communication sequence has the effect of *selecting columns* from $\mathcal{C}$.
- Can we always arrange matters so that I can select n *independent columns* over any consecutive k_f steps?

Suppose that A is invertible and the pair (A, B) is reachable. For any integer $1 \le w_\rho < m$, there exists an m-to-w_ρ communication sequence $\rho(\cdot)$ and an integer $k_f \le \left\lceil \frac{n}{w_\rho} \right\rceil \cdot n$, such that the plant is reachable in $[0, k_f]$.

Sequences that Preserve Reachability & Observability

- There exist integers i, N, and an N-periodic communication sequence under which the plant is reachable in $[i, i+l]$ for any i (l-step reachable).
  Constructive algorithm based on the matrix $\mathcal{C}$ and the number of available input channels, w_m [Zhang and Hristu-Varsakelis, ACC 05].

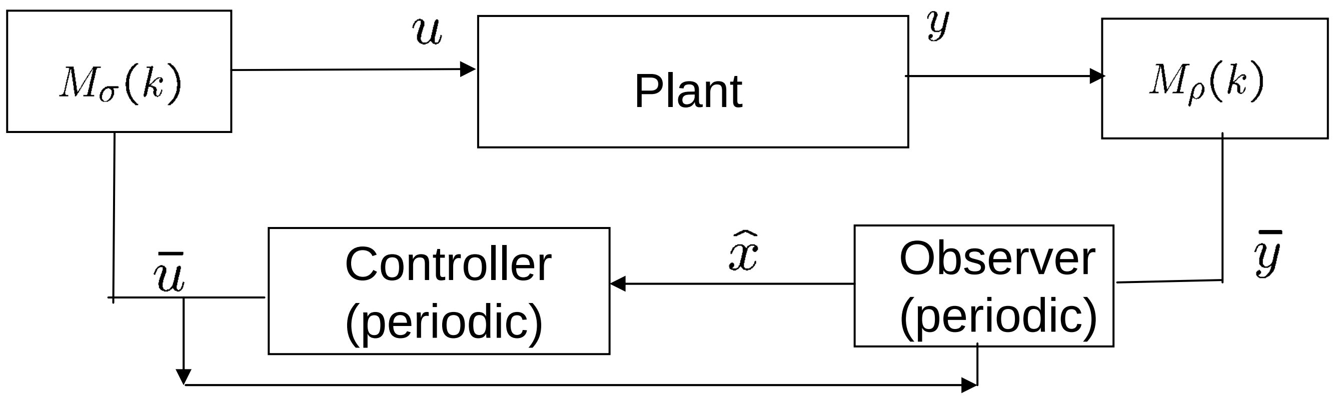
- A similar result holds for observability, this time finding a sequence that selects n independent rows from:

$$\mathcal{O} = \begin{bmatrix} M_\sigma(0)C \\ M_\sigma(1)CA \\ \vdots \\ M_\sigma(k_f-1)CA^{k_f-1} \end{bmatrix}$$

To stabilize a NCS, select periodic communication sequences $\rho(k)$, $\sigma(k)$ that preserve reachability and observability, then design a stabilizing feedback controller (and observer, if necessary) for the resulting periodic system. This is always possible if A invertible and (A,B) reachable.

Note: Invertibility of A is necessary: for $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$, there is no reachability-preserving 3-to-1 sequence.

Example – output feedback stabilization



$$A = \begin{bmatrix} 1 & 1/5 & 0 & 0 \\ 0 & 11/4 & 0 & 1/5 \\ 1 & 1/5 & 1/3 & 3/4 \\ 0 & -1 & 0 & 1/4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- We constructed 2-periodic input/output sequences $\{\sigma(0), \sigma(1), \cdots\} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \cdots \right\}$

$$\{\rho(0), \rho(1), \cdots\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \cdots \right\}$$

- Plant is 7-step reachable, 7-step observable.

- Designed a periodic observer and a periodic state feedback controller using standard linear theory (period-2 gain sequences).

Observer / Controller Design

Observer:

$$\hat{x}(k+1) = A\hat{x}(k) + \bar{B}(k)\bar{u}(k) + H(k)[\bar{y}(k) - \hat{y}(k)]$$
$$\hat{y}(k) = \bar{C}(k)\hat{x}(k)$$

Controller:

$$\bar{u}(k) = K(k)\hat{x}(k)$$

Gains:

$$K(k) = -\bar{B}^T(k)(A^{-1})^T \mathcal{W}_\alpha^{-1}(k, k+l)$$
$$H(k) = [(A^{-l})^T \mathcal{M}_\alpha(k-l+1, k+1) A^{-l}]^{-1}(A^{-1})^T \bar{C}^T(k)$$

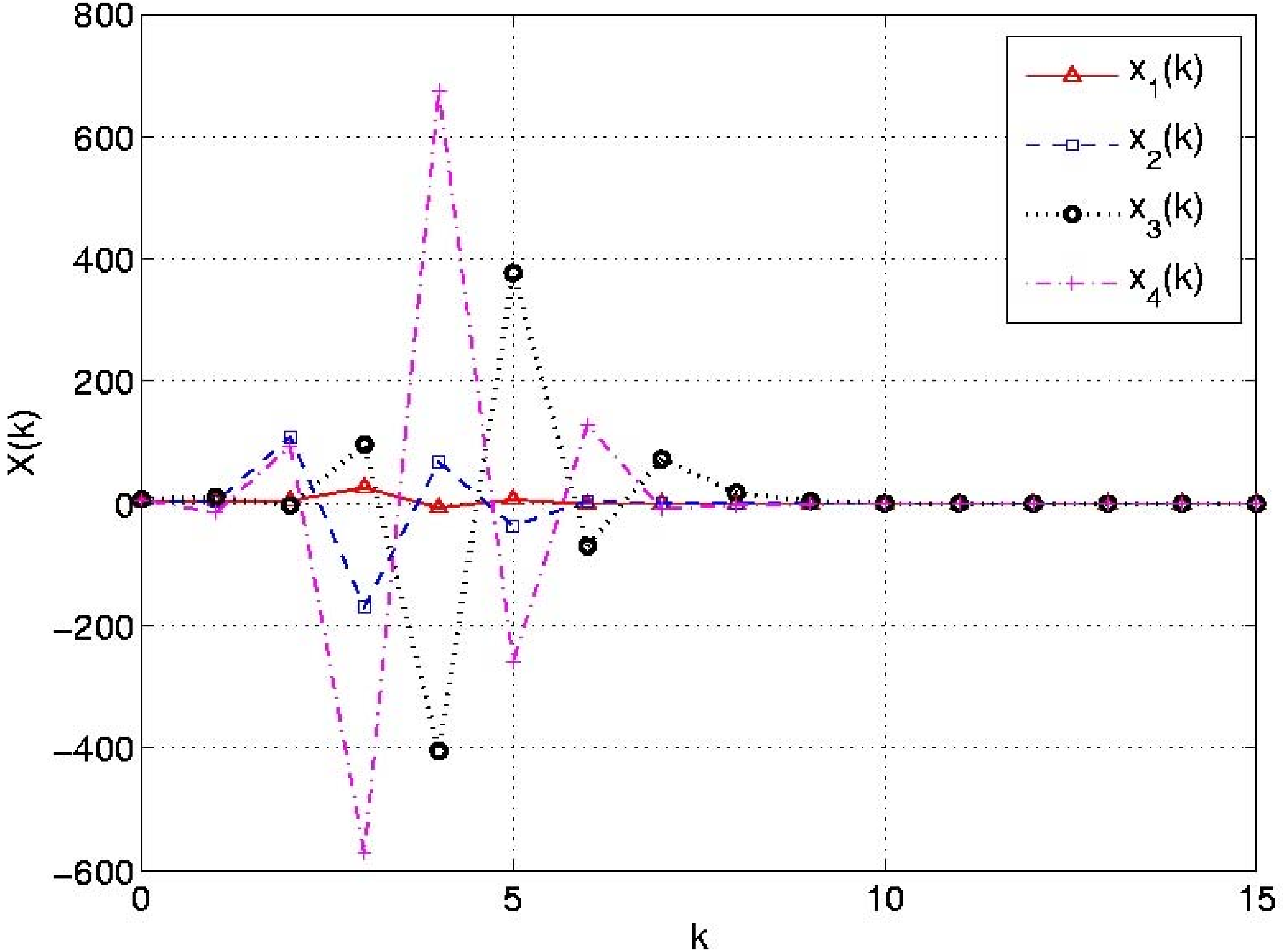
where

$$\mathcal{W}_\alpha(k_0, k_f) = \sum_{j=k_0}^{k_f-1} \alpha^{4(k_0-j)} A^{k_0-j-1} \bar{B}(j) \bar{B}^T(j) (A^{k_0-j-1})^T$$

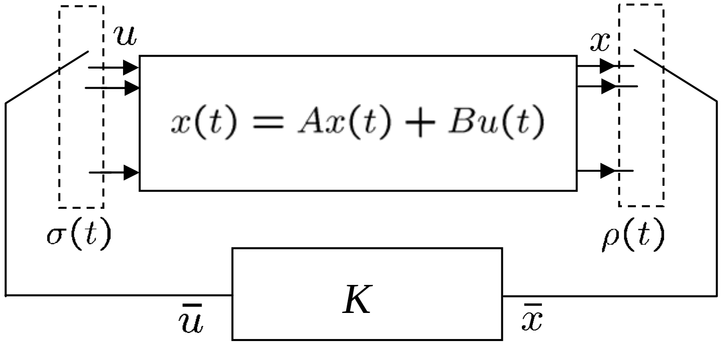
$$\mathcal{M}_\alpha(k_0, k_f) = \sum_{j=k_0}^{k_f-1} \alpha^{4(j-k_f+1)} (A^{j-k_0})^T \bar{C}^T(j) \bar{C}(j) A^{j-k_0}$$

$l=7$, and select $\alpha>1$ for closed-loop decay rate of $(1/\alpha)^k$

State Evolution



Stabilization under Feedback-based communication



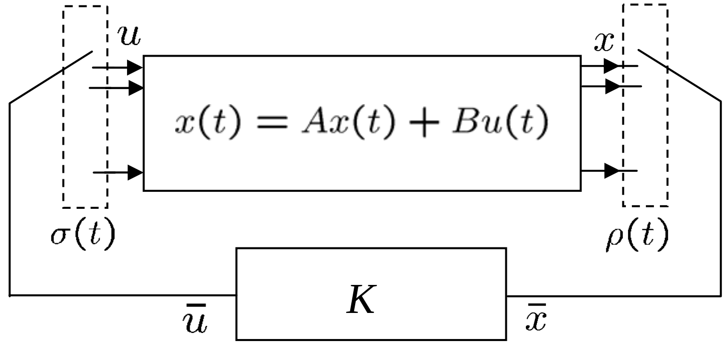
•In some cases, it could be advantageous to have the communication sequence depend on the states/outputs as opposed to being defined in advance.

In continuous time define an *M-to-N communication sequence as* $\sigma(t): R^+ \mapsto \{0,1\}^M$, satisfying $||\sigma(t)||^2 = N, \forall t.$

- Let controller be static state feedack: $\bar{u}(t) = K\bar{x}(t)$

•Goal: Design the controller *K* and a feedback-based policy $\sigma = \sigma(x,t)$ in order to stabilize the closed-look system

Feedback-based communication cont'd



- Again, forgo any ZOH and instead use *zero* for any sensor/actuator readings that are unavailable due to limited communication

- Ignoring "inactive" sensors and actuators gives rise to *switched dynamics* for the closed-loop system:

$u(t) = M_\rho(t)\bar{u}(t)$ and $\bar{x}(t) = M_\sigma(t) \cdot x(t)$

so that
$$\dot{x}(t) = \underbrace{(A + BM_\rho(t)KM_\sigma(t))}_{\widehat{A}(t)}x(t)$$

where again $M_\rho(t) \triangleq diag(\rho(t)),\ M_\sigma(t) \triangleq diag(\sigma(t))$

Feedback-based communication cont'd

- Wlog, suppose that only 1 sensor and 1 actuator can communicate with the controller at any time ($w_p = w_m = 1$).

- There are $n.m$ possible closed-loop dynamics: $\dot{x} = \hat{A}(t)x$

$$\hat{A}(t) \in \{\mathcal{A}_{ij} : i = 1, \cdots, m;\ j = 1, \cdots, n\}$$ where:

$$\mathcal{A}_{ij} = A + BK_{ij}$$ and $K_{ij} = diag(e^i_m) \cdot K \cdot diag(e^j_n)$, with

$e^i_m,\ e^j_n$: standard basis vectors in R^m, R^n.

Idea:

1. Design the gain K so that the $\mathcal{A}_{ij}$ have a stable convex combination, say

$$\mathcal{A} = \sum \alpha_{ij}\mathcal{A}_{ij}, \quad \sum \alpha_{ij} = 1, \quad \alpha_{ij} \geq 0$$

2. Then, at any time, at least one of the $n.m$ communication choices leads to the decrease of a Lyapunov function $x^T(t)Px(t)$, i.e.

$$x^T(\mathcal{A}^T_{ij}P + P\mathcal{A}_{ij})x < 0$$ for some (i,j)

This defines a stabilizing communication policy!

Feedback-based communication cont'd

Computing stabilizing a gain & communication policy:

1. Choose any α_{ij} so that $\sum \alpha_{ij} = 1, \quad \alpha_{ij} \geq 0$. Think of these as the relative amount of attention to be given by the controller to the *i-th* sensor and *j-th* actuator.

2. Choose G to place the eigenvalues of $\mathcal{A} = \sum \alpha_{ij} \mathcal{A}_{ij} = A + BG$

3. Solve for the controller gain K from $K_{ij} = \alpha_{ij} G_{ij}$

Possible communication policies:

Denote the switching signal by $s(t)=(i(t),j(t))$:

1. *Fastest Decay* (FD): $s(t) = arg \min_{i,j} x^T(t)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]x(t)$

2. *Weighted Fastest Decay* (WFD): $s(t) = arg \min_{i,j} \alpha_{ij} x^T(t)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]x(t)$

where $P=P^T>0$ satisfies $\mathcal{A}^T P + P\mathcal{A} = -Q, \quad Q = Q^T > 0$

Stability under the FD and WFD policies

Fastest Decay (FD): $s(t) = arg\min_{i,j} x^T(t)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]x(t)$

Weighted Fastest Decay (WFD): $s(t) = arg\min_{i,j} \alpha_{ij} x^T(t)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]x(t)$

Quadratic Stability under FD / WFD:

Choose α_{ij}, K, P as suggested. Then, under the FD or WFD policy, all trajectories of the closed-loop system satisfy $\frac{d}{dt}V(x) \leq -\epsilon x^T x$, where $\epsilon \geq \epsilon^* = \frac{\lambda_{min}(Q)}{m \cdot n \cdot \alpha_{max}}$

(because there is always a choice of $s(t)=(i(t),j(t))$ for which V decreases.)

Stability without chattering

FD / WFD can lead to chattering. To avoid this, try to impose a *minimum dwell time:*

Modified FD

0. Choose ε_0, such that $0 < \epsilon_0 < \epsilon^* = \frac{\lambda_{min}(Q)}{m \cdot n \cdot \alpha_{max}}$

1. At $t=t_0$, let $s(t_0) = arg \min_{i,j} x^T(t_0)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]x(t_0)$ *(FD policy)*

2. Let $s(t)=s(t_k)$ on $[t_k, t_{k+1})$, where*:*

$$t_{k+1} = \inf_{t>t_0} \{t : x^T(t_0)[\mathcal{A}_{s(t_0)}^T P + P\mathcal{A}_{s(t_0)}]x(t_0) \geq -\epsilon_0 x^T(t)x(t)\}$$

Quadratic Stability under Modified FD:

Choose α_{ij}, K, P as before. Then, under the *Modified FD* policy, all trajectories of the closed-loop system satisfy $\frac{d}{dt}V(x) \leq -\epsilon_0 x^T x$.

Moreover, the switching rate is bounded by $1/\tau$, where $\tau \sim (\epsilon^* - \epsilon_0)$

(because at t_k, the decay rate of $\frac{\dot{V}(t)}{x^T x}$ is at least ϵ^* (by FD), and $\frac{d}{dt}\frac{\dot{V}(t)}{x^T x}$ is bounded above)

Example 1: Using the FD policy

$$\dot{x} = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 5.67 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix} u$$

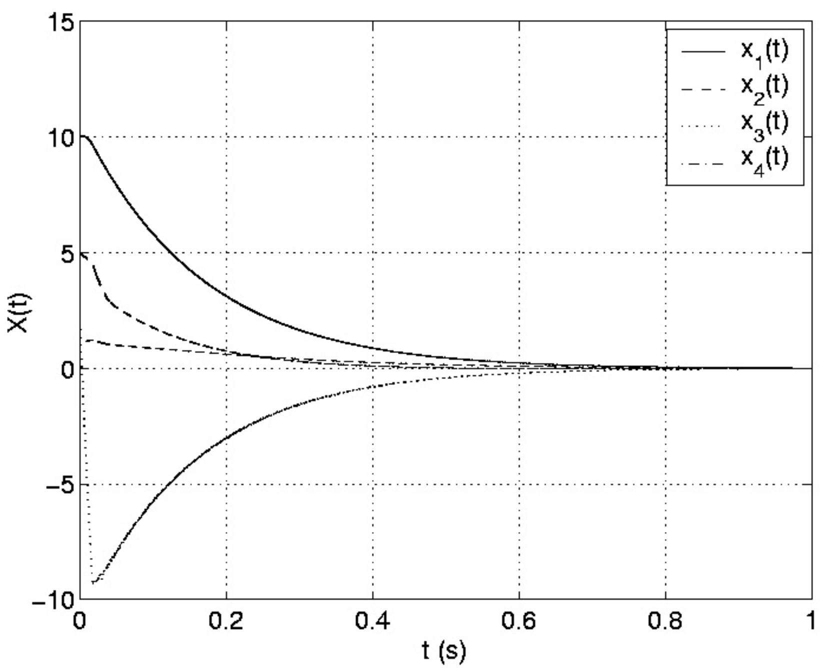
•Only *1 of 4* sensors and *1 of 2* actuators can communicate with the controller at any one time (i.e. $w_p = w_m = 1$.)

•We chose to place the eigenvalues of $A+BG$ at: $(-5,-6,-4,-3)$

For a choice of $[\alpha_{ij}] = \begin{bmatrix} 1/6 & 1/6 & 1/6 & 1/6 \\ 1/12 & 1/12 & 1/12 & 1/12 \end{bmatrix}$ we obtained the controller gains:

$$K = \begin{bmatrix} 0.5463 & -3.1950 & -0.8567 & -2.2001 \\ 23.0186 & 4.3389 & 10.4101 & -2.6616 \end{bmatrix}$$

State Evolution under FD

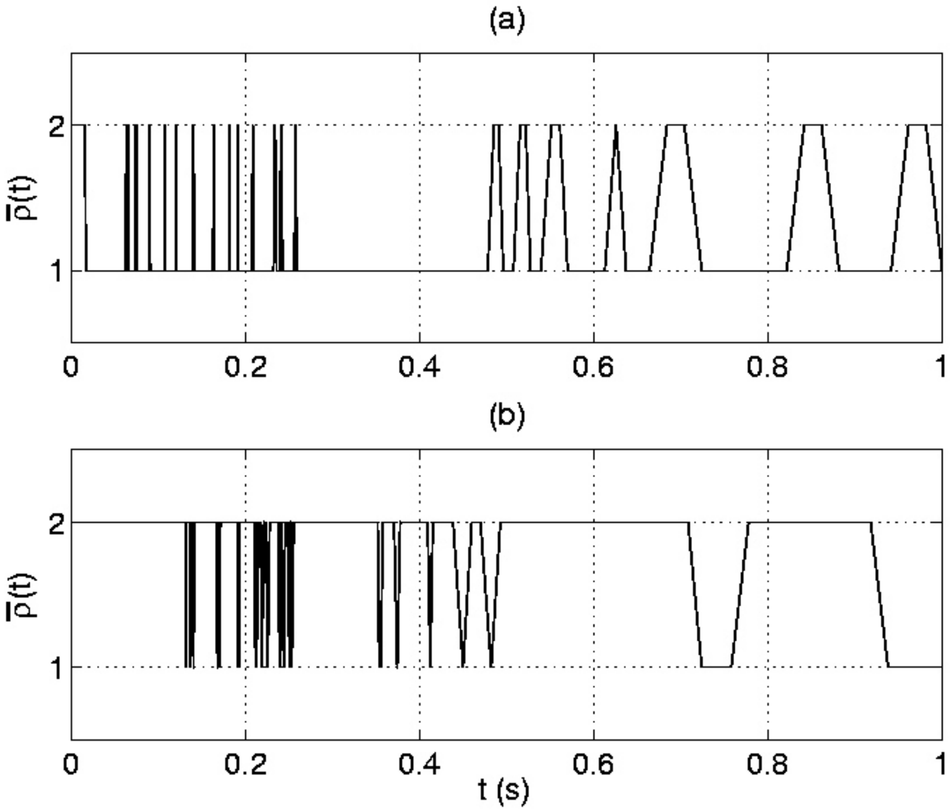


Example 1: Time vs. Control effort

$$[\alpha_{ij}] = \begin{bmatrix} 1/6 & 1/6 & 1/6 & 1/6 \\ 1/12 & 1/12 & 1/12 & 1/12 \end{bmatrix} \Longrightarrow K_a = \begin{bmatrix} 0.5463 & -3.1950 & -0.8567 & -2.2001 \\ 23.0186 & 4.3389 & 10.4101 & -2.6616 \end{bmatrix}$$

$$[\alpha_{ij}] = \begin{bmatrix} 1/12 & 1/12 & 1/12 & 1/12 \\ 1/6 & 1/6 & 1/6 & 1/6 \end{bmatrix} \Longrightarrow K_b = \begin{bmatrix} 1.0927 & -6.3900 & -1.7135 & -4.4002 \\ 11.5093 & 2.1694 & 5.2050 & -1.3308] \end{bmatrix}$$

Notice how the amount of time spent communicating with each actuator can be manipulated by choosing the weights α_{ij}



Optimizing the controller

•We can choose the weights α_{ij} to achieve various objectives while maintaining stability.

Example: avoiding actuator saturation

- Suppose that $|u(t)_i| < M_i$ for all $i=1,\ldots,m$
- There is a maximum radius r_{max} such that all closed-loop trajectories starting with $||x_0(0)||<r$ are guaranteed not to saturate the actuators.
- Bound for r_{max} depends on M, K, and $\lambda_{max}(P)/\lambda_{min}(P)$.
- K depends on α_{ij} – choose them to maximize r_{max} (its upper bound)

To maximize our upper bound for r_{max}

1. Choose P to minimize $\lambda_{max}(P)/\lambda_{min}(P)$, and/or
2. Scale the rows of K to maximize the region in which we are guaranteed not to saturate, by a factor of β_i, where:

$$\forall i,j: \quad \beta_i \frac{M_i}{||K'_i||/n} = \beta_j \frac{M_j}{||K'_j||/n}, \quad i,j = 1,\ldots,m.$$

where K'_i are the rows of the gain matrix K' obtained when $\alpha_{ij}=1/n$ and $\alpha_{ij}=\beta_i/n \;\forall i=1,\ldots,m,\; j=1,\ldots,n$.

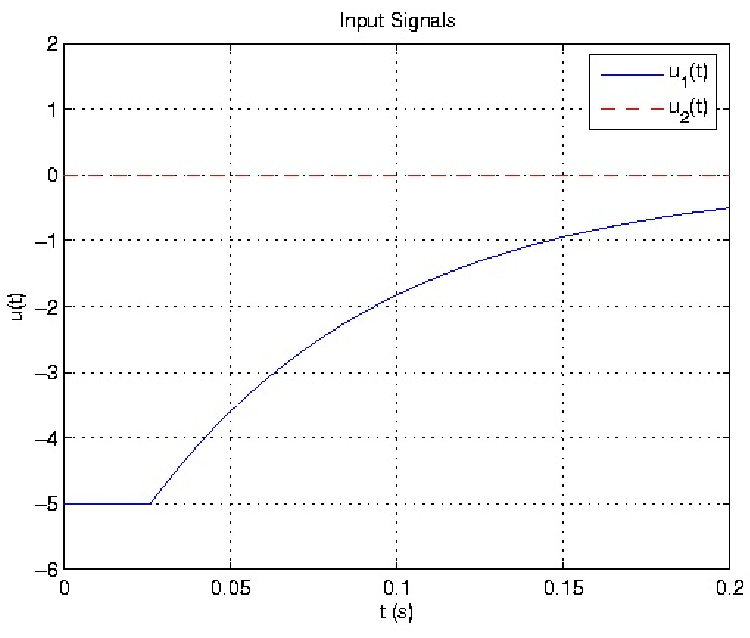
Example – Avoiding saturation by choice of α_{ij}

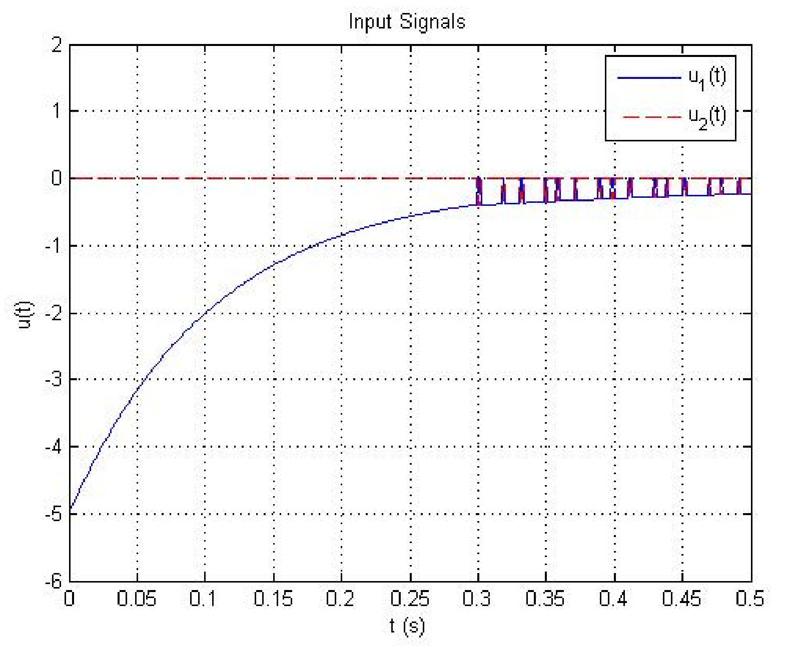
$$\dot{x} = \begin{bmatrix} 3 & 1 \\ 2 & -1.5 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u; \qquad x(0) = [0.4, 0]^T$$

- Set $P=I$, and saturation bounds $M_1=M_2=5$. Apply Modified FD policy.
- Place eigenvalues of $A+BG$ at $[-1, -1.5]$

- Weights: $\alpha_1 = \alpha_2 = 0.5.$ $\alpha_1^* = 0.6883, \ \alpha_2^* = 0.3117.$

- Gain: $K = \begin{bmatrix} -16.8 & -1.6 \\ -7.6 & 0.8 \end{bmatrix}$ $K = \begin{bmatrix} -12.21 & -1.16 \\ -12.19 & 1.28 \end{bmatrix}$





Summary & Conclusions

- Stabilization of Networked Control Systems (NCS)

- Reduced complexity of the stabilization problem by "ignoring" sensors/actuators that are not actively communicating, and by forgoing ZOH.

- The approach makes the selection of controllers and communication sequences tractable and straightforward, compared to when ZOH is used.

- "Static" communication sequences
  - Algorithm for selecting periodic sequences that preserve stabilizability.
  - Controller design using known state-space methods.

- Feedback-based communication sequences
  - An algorithm for jointly selecting the controller and communication.
  - Trade-off between time and control effort.