A BEHAVIORAL APPROACH TO H_∞ CONTROL

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ABSTRACT. The purpose of this talk is to outline an approach to the synthesis of controllers (and filters) in the behavioral framework. The main advantage is that the results obtained this way are 'representation independent', with all the algorithmic advantages that this entails. The aim is to explain the solution of the problem discussed below, but, before doing so, a number of background concepts need to be introduced.

We have a linear time-invariant plant in which to-be-controlled variables $w = (d, z)$ (with d exogenous disturbances and z endogenous to-be-controlled variables) should be regulated by attaching a controller that acts on control variables c. The basic control specifications are expressed by a quadratic functional $|w|_S^2$, which here equals $|d|^2 - |z|^2$, whose integral needs to be non-negative. Let $\mathcal{P}$ be the plant behavior, the behavior of the variables w before the controller is attached, and $\mathcal{N}$ is the sub-behavior compatible with $c = 0$.

This leads to the following mathematical problem. Let $\mathcal{N}, \mathcal{P}$ be given behaviors, with $\mathcal{N} \subset \mathcal{P}$; Find a behavior $\mathcal{K}$ (the controlled behavior) such that

1. $\mathcal{N} \subset \mathcal{K} \subset \mathcal{P}$ (realizability and implementability),
2. $\mathcal{K}$ is Σ–dissipative on $\mathbb{R}_-$ (disturbance attenuation and stability),
3. The disturbances are free in $\mathcal{K}$ (liveness).

We will explain how this problem is related to the standard H_∞ and robust control. The condition $\mathcal{N} \subset \mathcal{K}$ formalizes the fact that the controller must act through the control variables. It is fully analogous to the special case of feedback control, in which the controller processes the measured outputs in order to compute the actuator inputs. Two important special cases that will be given particular attention are: the case $\mathcal{N} = 0$ that corresponds to 'full information control', and the case that corresponds to H_∞-filtering, in which the plant behavior $\mathcal{P}$ is unrestricted.

The problem formulated above admits a very elegant answer. The controlled behavior $\mathcal{K}$ exists if and only if

1. $\mathcal{N}$ is Σ – dissipative on $\mathbb{R}_-$,
2. $\mathcal{P}^\perp$ is $-\Sigma^{-1}$–dissipative on $\mathbb{R}_+$,
3. A subtle coupling condition on the storage functions of $\mathcal{N}$ and $\mathcal{P}^\perp$ holds.

What this last-mentioned condition is, will be disclosed during the seminar.