Multiuser Estimation

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Outline of the Talk

- Estimation using sensor data: Problem Statement and Brief Description of Possible Solutions
- Estimation using Optimum Detection (Ignoring Correlated Sources)
  - Multiuser Detection
- Multiuser Estimation
  - System Description
  - Approximate Nonlinear Filtering
- Simulations
- Conclusions and Future Directions

- **Problem**: Data from different sensors is correlated.
  - Sending the correlated data without data compression is wasting the precious bandwidth.
  - The data is distributed between different sensors.
- **Solution**:Use distributed data compression [Slepian-Wolf 1973].
  - Requirement:Nodes should know the probability distribution function of the measurement in neighboring nodes.
  - Unknown territory: Distributed compression for partially observed Markov processes.

D. Slepian and J. K. Wolf, "Noiseless Coding of Correlated Information Sources", IEEE Trans. Inform. Theory, pp.471-480 July 1973.

S. Pradhan et al, "Distributed Compression in a Dense Microsensor Network", IEEE Signal Processing Magazine, pp. 51-60 March 2002.

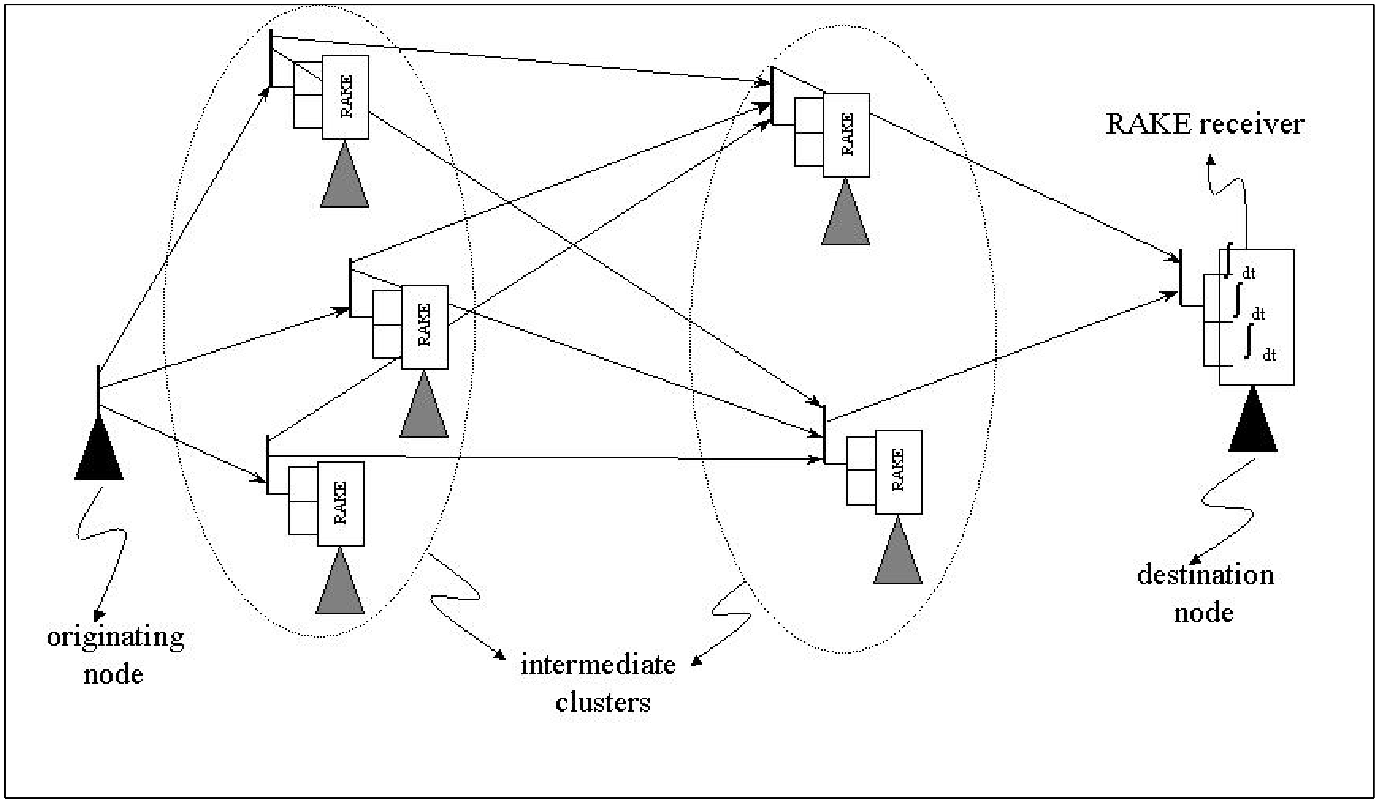
Alternative Solutions

- **Are there other solutions to transmit the correlated sensor data?** YES.
- **Cooperative Diversity:** Use correlated data in communication scheme [Mercado Azimi-Sadjadi 2003].
- **Multiuser Estimation:** Use correlated data to design the optimum receiver [Azimi-Sadjadi 2003].

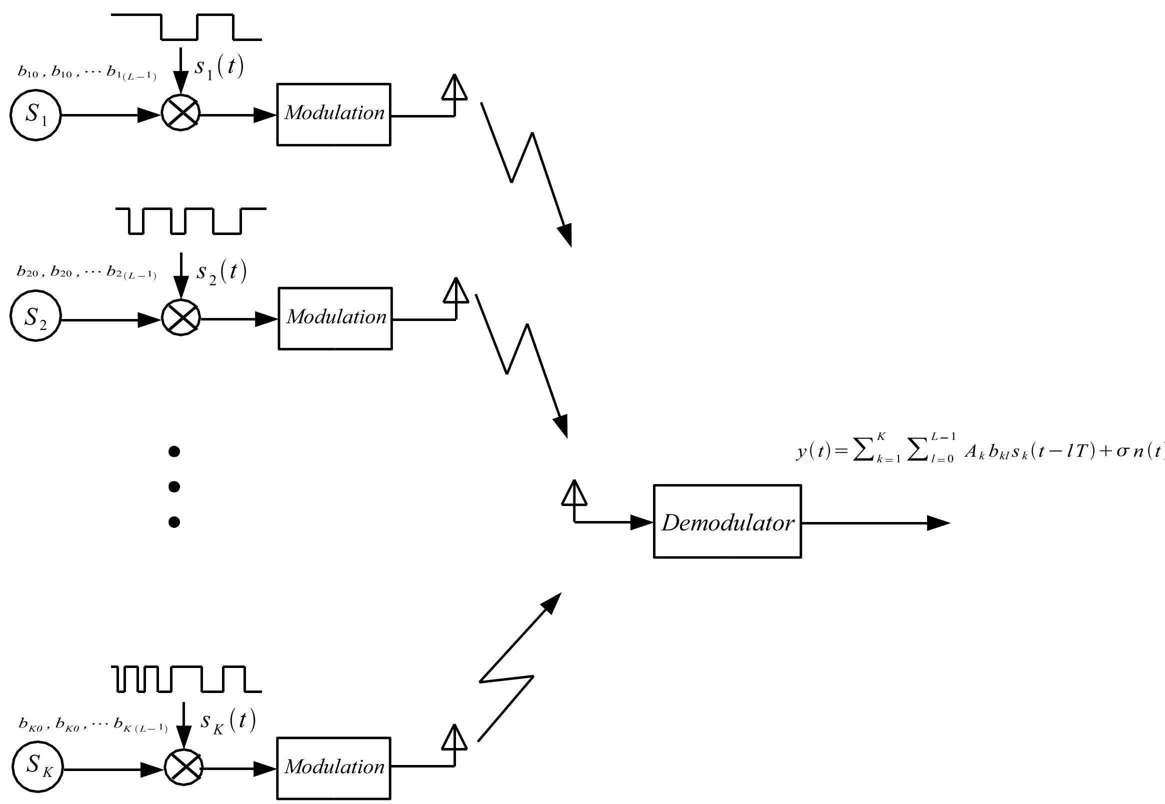
A. Mercado and B. Azimi-Sadjadi, "Power Efficient Link for Multi-Hop Wireless Networks, 41st Annual Allerton Conference on Communication, Control, and Computing, October 2003.

B. Azimi-Sadjadi, "Multiuser Estimation", submitted.

Cooperative Diversity



Estimator with optimum detector



K-user synchronous CDMA system

Estimator with optimum detector cont.

multiuser detection

For $L = 1$, the received signal is:

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t), \quad t \in [0, T] \tag{1}$$

- $1/T$ is the data rate.
- $s_k(t)$ is the spreading code assigned to the k^{th} user, normalized so as to have unit energy
  $$\|s_k\|^2 = \int_0^T s_k^2(t)dt = 1.$$
  $s_k(t)$ is assumed to be zero outside the interval $[0, T]$.
- A_k is the received amplitude of the k^{th} user's signal.
- $b_k \in \{-1, 1\}$ is the bit transmitted by the k^{th} user.
- $n(t)$ is white Gaussian noise with unit power spectral density.

Estimator with optimum detector cont.

multiuser detection cont.

Define

$$\rho_{ij} = \int_0^T s_i(t)s_j(t)dt$$
$$n_k = \sigma \int_0^T n(t)s_k(t)dt$$
$$y_k = \int_0^T y(t)s_k(t)dt$$

$$\mathbf{y} = R\mathcal{A}\mathbf{b} + \mathbf{n}, \tag{2}$$

where $R = \{\rho_{ij}\}$ is the cross correlation matrix, $\mathbf{y} = [y_1, \cdots, y_K]'$, $\mathcal{A} = \text{diag}(A_1, \cdots, A_K)$, $\mathbf{b} = [b_1, \cdots, b_K]'$, and $\mathbf{n} = [n_1, \cdots, n_K]'$.

Estimator with optimum detector cont.

multiuser detection cont.

For a K-user CDMA channel, an optimal receiver chooses the $\mathbf{b} = [b_1, \cdots, b_K]'$ that maximizes

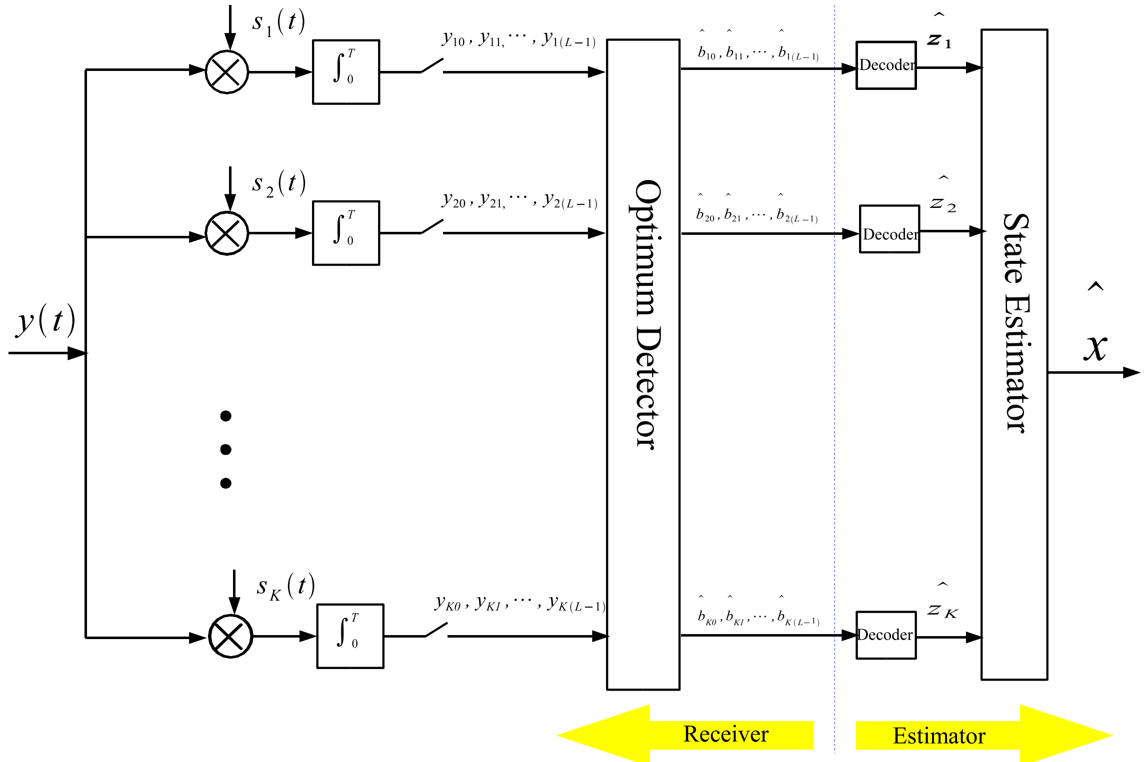
$$\exp\left(-\frac{1}{2\sigma^2}\int_0^T [y(t) - \sum_{k=1}^{K} b_k A_k s_k(t)]^2 dt\right),$$

or equivalently, maximizes (since $\mathbf{y}$ is a sufficient statistic for $\mathbf{b}$)

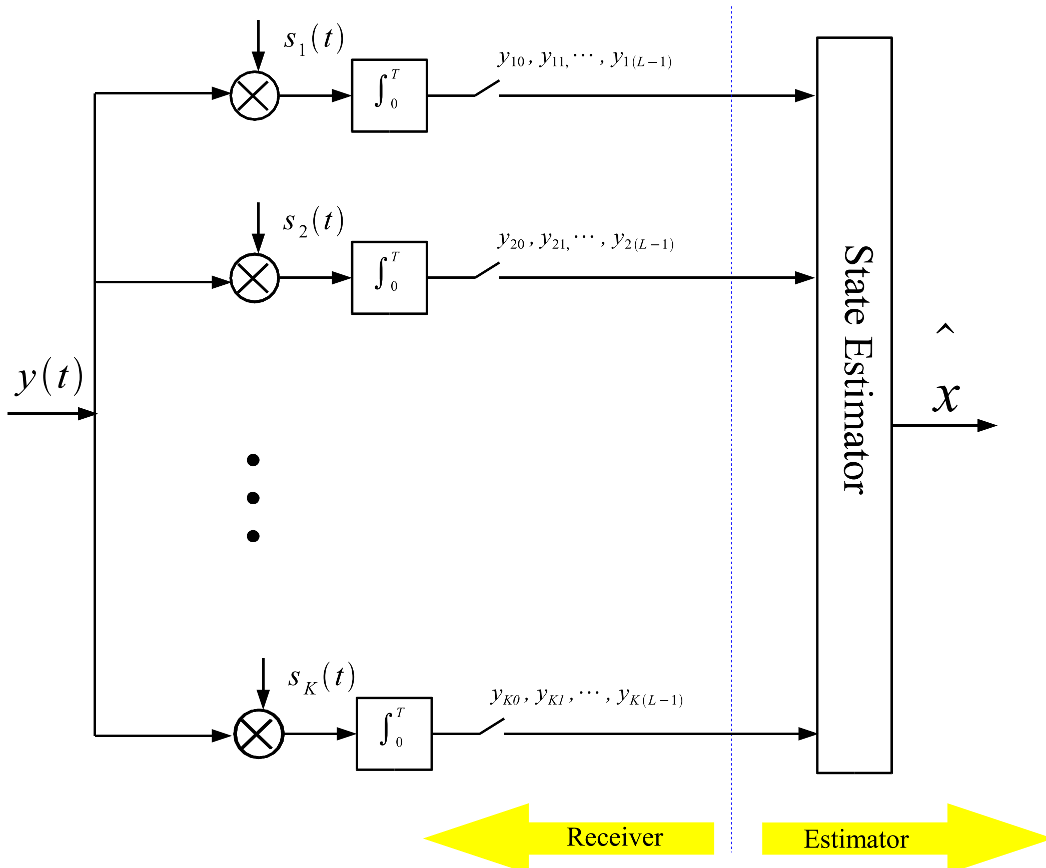
$$\Omega(\mathbf{b}) \quad = \quad 2\mathbf{b}'\mathcal{A}\mathbf{y} - \mathbf{b}'\mathcal{H}\mathbf{b}, \tag{3}$$

where $\mathcal{H} = \mathcal{A}R\mathcal{A}$.

Estimator with optimum detector cont.



Multiuser Estimation



Multiuser Estimation Cont.

- **Difficulties:** Due to quantization Multiuser Estimation is nonlinear even if the system is linear.
- **Solution:** Approximate Nonlinear Filtering.

System Equations

$$\begin{aligned}
\mathbf{x}(i+1) &= \mathbf{f}(\mathbf{x}(i), \mathbf{w}(i)) \\
\mathbf{z}(i) &= \mathbf{g}(\mathbf{x}(i), \mathbf{v}(i)) \\
B(i) &= Q(\mathbf{z}(i)) \\
Y(i) &= R\mathcal{A}B(i) + N(i).
\end{aligned} \tag{4}$$

- $\mathbf{x}(i) \in \mathcal{R}^n$ is a random process to be estimated by the multiuser estimator.
- $\mathbf{z}(i) = [\mathbf{z}_1'(i), \cdots, \mathbf{z}_K'(i)]'$ where $\mathbf{z}_k(i)$ is the output of the k^{th} sensor and it is a function of the random process $\mathbf{x}(i)$, $\mathbf{z}_k(i) = \mathbf{g}_k(\mathbf{x}(i), \mathbf{v}_k(i))$, where $\mathbf{v}_k(i)$ is the observation noise.

Nonlinear Filtering

- Step 1 . Initialization:

$$p_0(\mathbf{x}_0|\mathbf{y}_0) = p(\mathbf{x}_0).$$

- Step 2 . Diffusion:

$$p_{(i+1)^-}(\mathbf{x}_{i+1}|\mathcal{Y}_i) = \int p(\mathbf{x}_{i+1}|\mathbf{x}_i) p_i(\mathbf{x}_i|\mathcal{Y}_i) d\mathbf{x}_i,$$

where $\mathcal{Y}_i = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_i\}$.

- Step 3 . Bayes' rule update:

$$p_{(i+1)}(\mathbf{x}_{i+1}|\mathcal{Y}_{i+1}) = \frac{p(\mathbf{y}_{i+1}|\mathbf{x}_{i+1}) p_{(i+1)^-}(\mathbf{x}_{i+1}|\mathcal{Y}_i)}{\int p(\mathbf{y}_{i+1}|\mathbf{x}_{i+1}) p_{(i+1)^-}(\mathbf{x}_{i+1}|\mathcal{Y}_i) d\mathbf{x}_{i+1}} \quad ,$$

- Step 4 . $i \leftarrow i + 1$; go to Step (2).

$$p_{(i+1)}(\mathbf{x}_{i+1}|Y_0^{i+1}) = \frac{p(Y(i+1)|\mathbf{x}(i+1))p_{(i+1)^-}(\mathbf{x}(i+1)|Y_0^i)}{\int p(Y(i+1)|\mathbf{x}(i+1))p_{(i+1)^-}(\mathbf{x}(i+1)|Y_0^i)d\mathbf{x}_{i+1}} \ ,$$

where $p(Y(i+1)|\mathbf{x}(i+1))$ can be expressed as follows:

$$\begin{aligned} p(Y(i+1)|\mathbf{x}(i+1)) &= \int_{\mathbf{z}} p(Y(i+1)|\mathbf{z})p(\mathbf{z}|\mathbf{x}(i+1))d\mathbf{z} \\ &= \int_{\mathbf{z}} p(Y(i+1)|Q(\mathbf{z}))p(\mathbf{z}|\mathbf{x}(i+1))d\mathbf{z} \end{aligned} \ .$$

Algorithm 1 *Modified Particle Filtering*

- *Step 1 . Initialization*
  - ◇ *Sample* $\mathbf{x}_0^1, \cdots, \mathbf{x}_0^N$, *N i.i.d. random vectors with the distribution* $P_0(\mathbf{x})$.

- *Step 2 . Diffusion*
  - ◇ *Find* $\hat{\mathbf{x}}_{i+1}^1, \cdots, \hat{\mathbf{x}}_{i+1}^N$ *from the given* $\mathbf{x}_i^1, \cdots, \mathbf{x}_i^N$, *using the dynamic rules:*

$$\mathbf{x}(i+1) = \mathbf{f}(\mathbf{x}(i), \mathbf{w}(i)).$$

- *Step 3. Approximate* $p(Y(i+1)|\hat{\mathbf{x}}_{i+1}^j)$, *for* $j = 1, \cdots, N$.

  1- *Set* $j = 1$.

  2- *Generate* $\mathbf{v}_1, \cdots, \mathbf{v}_M$, *M i.i.d. random vectors according to the density of the observation noise. Use these random variable,* $\mathbf{x}_{i+1}^j$, *and the observation equation to generate* $\mathbf{z}_1, \cdots, \mathbf{z}_M$.

  3- *Set* $B_l = Q(\mathbf{z}_l)$ *for* $l = 1, \cdots, M$.

4- Approximate $p(Y(i+1)|\hat{\mathbf{x}}^j_{i+1})$ *as follows:*

$$\hat{p}(Y(i+1)|\hat{\mathbf{x}}^j_{i+1}) = \sum_{l=1}^{M} p(Y(i+1)|B_l)p(\mathbf{z}_l|\hat{\mathbf{x}}^j_{i+1})$$

5- $j = j+1$ *go to 2.*

- *Step 4 . Use Bayes' Rule*

$$P^N_{(i+1)}(\mathbf{x}) = \frac{\sum_{j=1}^{N} \delta_{\hat{\mathbf{x}}^j_{i+1}}(\mathbf{x}) \cdot \hat{p}(Y(i+1)|\hat{\mathbf{x}}^j_{i+1})}{\sum_{j=1}^{N} \hat{p}(Y(i+1)|\hat{\mathbf{x}}^j_{i+1}))}$$

- *Step 5 . Resample*
  - ◇ *Sample* $\mathbf{x}^1_{i+1}, \cdots, \mathbf{x}^N_{i+1}$ *according to* $P^N_{(i+1)}(\mathbf{x})$
- *Step 6 .* $i \leftarrow i+1$; *go to Step (2).*

Simulation: A two sensor example

$$\begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}(i+1) = \begin{pmatrix} 1 & 0 & .1 & 0 \\ 0 & 1 & 0 & .1 \\ 0 & 0 & 0.4 & 0.86 \\ 0 & 0 & -0.86 & 0.4 \end{pmatrix} \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}(i) + \alpha \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix}(i)$$

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}(i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}(i) + \beta \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}(i),$$

where the noise vectors $(w_1, w_2, w_3, w_4)'$ and $(v_1, v_2)'$ are Gaussian with zero mean and unit variance.

- Two methods are compared: Kalman Filter and Multiuser Estimator
- The observation for the Kalman Filter is $\hat{\mathbf{z}}$.
- The observation for Multiuser Estimator is $Y(i)$.

Experiment	α, β, σ, L	Error for Kalman filter	Error for multiuser estimator
1	0.1, , 0.1, , 1, 6	2.41	0.38
2	0.1, , 0.1, , 0.5, 6	0.70	0.17
3	0.3, , 0.3, , 0.5, 6	0.78	0.34
4	0.1, , 0.1, , 1, 9	2.24	0.37

Table 1: Comparison of the errors between the multiuser estimator and the Kalman filter.

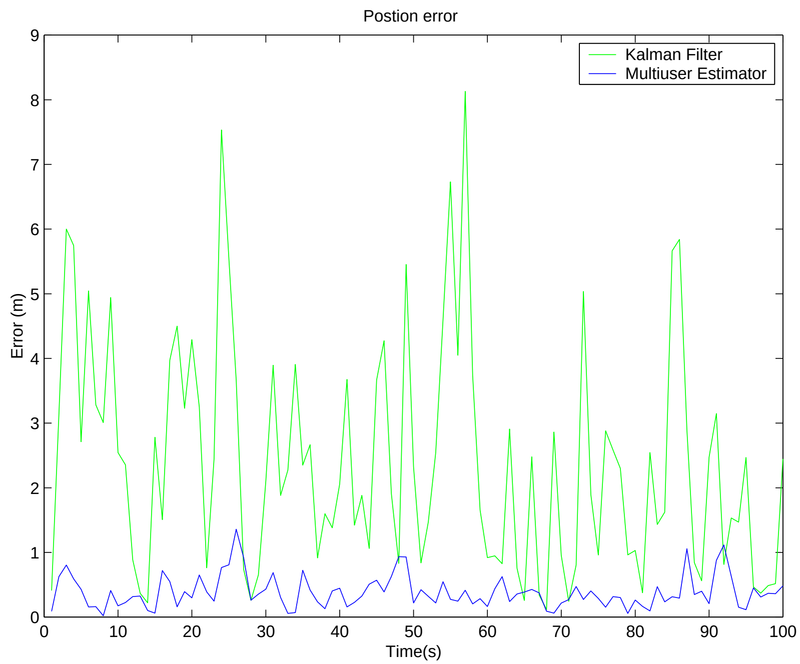


Figure 1: Comparison of the position error for system (5). In this experiment $\sigma = 1$, $\alpha = \beta = 0.1$, and the number of bits for each sensor measurement is 6 bits.

Conclusions and Future Direction

- The correlation in sensor data should be used in the optimum receiver to achieve better performance.
- In multiuser estimation the main goal is to find the estimate of the state $\mathbf{x}(i)$ and the estimated transmitted bits are only the bi-product of this technique.
- In this talk the sensors directly transmit to the final receiver. In the case of multi-hop networks we can use the correlation between different sources for an additional "error correcting" power.
- In a wireless network the correlation of different sources can be used in the communication scheme.