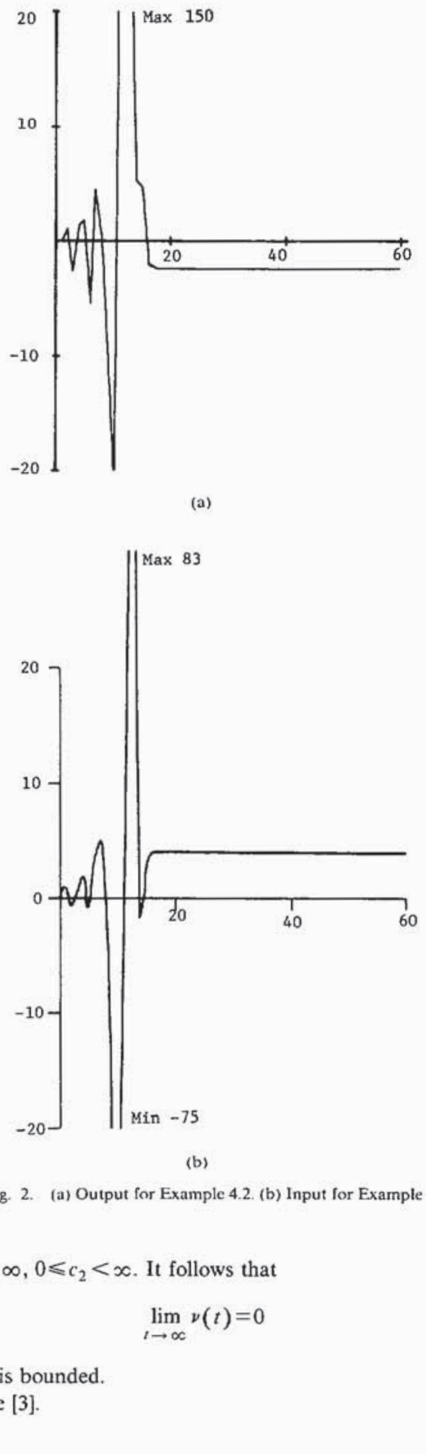

Fig. 2. (a) Output for Example 4.2. (b) Input for Example 4.2.

where $0 \leq c_1 < \infty$, $0 \leq c_2 < \infty$. It follows that

$$\lim_{t \to \infty} \nu(t) = 0 \tag{A.8}$$

and $\{\|\phi(t)\|\}$ is bounded.

Proof: See [3].

References

[1] A. Feuer and S. Morse, "Adaptive control of single-input single-output linear systems," *IEEE Trans. Automat. Contr.*, vol. AC-23, pp. 557–570, Aug. 1978.
[2] K. S. Narendra and L. S. Valavani, "Stable adaptive controller design-direct control," *IEEE Trans. Automat. Contr.*, vol. AC-23, pp. 570–583, Aug. 1978.
[3] G. C. Goodwin, P. J. Ramadge, and P. E. Caines, "Discrete time multivariable adaptive control," *IEEE Trans. Automat. Contr.* vol. AC-25, June, 1980.
[4] B. Egardt, "Stability of model reference adaptive and self tuning regulators," Dep. of Automat. Contr., Lund Inst. of Technol., Tech. Rep., Dec. 1978.
[5] A. S. Morse, "Global stability of parameters adaptive control systems," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 433–440, June 1980.
[6] K. S. Narendra and Y. H. Lin, "Stable discrete adaptive control," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 456–461, June 1980.
[7] G. C. Goodwin, P. J. Ramadge, and P. E. Caines, "Discrete time stochastic adaptive control," *SIAM J. Contr.*, 1981, to be published.
[8] G. C. Goodwin and K. S. Sin, "Stochastic adaptive control using a modified least squares algorithm," Dep. Elec. Eng., Univ. of Newcastle, Tech. Rep. EE7907, June 1979.
[9] K. J. Astrom and B. Wittenmark, "Analysis of a self tuning regulator for nonminimum phase systems," in *Proc. IFAC Symp. on Stoch. Contr.*, Budapest, Hungary, 1974.
[10] W. Wolovich, *Linear Multivariable Systems*. New York: Springer Verlag, 1974.
[11] W. R. E. Wouters, "Adaptive pole placement for linear stochastic systems with unknown parameters," in *Proc. IEEE CDC Conf.*, New Orleans, LA, 1977, pp. 159–166.
[12] N. T. Hung and B. D. O. Anderson, "Triangularization technique for the design of multivariable control systems," *IEEE Trans. Automat. Contr.*, vol. AC-24, pp. 455–460, June 1979.
[13] G. C. Goodwin and K. S. Sin, "Effect of model, system, and controller order on adaptive control," presented at *18th CDC Conf.*, FL, Dec. 1979.
[14] P. E. Wellstead, D. Prager and P. Zanker, "A pole assignment self tuning regulator," *UMIST Contr. Syst. Center*, Manchester, England, Rep. 434, Mar. 1979.
[15] H. Elliott and W. A. Wolovich, "Paramater adaptive identification and control," *IEEE Trans. Automat. Contr.*, vol AC-24, pp. 592–599, Aug. 1979.
[16] R. Kumar and J. B. Moore, "Convergence of adaptive minimum, variance algorithms via weighting coefficient selection," Dep. Elec. Eng., Univ. of Newcastle, Tech. Rep. 7917.
[17] G. Kreisselmeier, "Adaptive control via adaptive observation and asymptotic feedback matrix synthesis," *IEEE Trans. Automat. Contr.*, vol. AC-25, Aug. 1980.

Radon Inversion and Kalman Reconstructions: A Comparison

DAVID ROHLER AND P. S. KRISHNAPRASAD

***Abstract*— Some of the recent applications of computerized tomography (CT) such as gated cardiac scanning and dynamic scanning are challenging the capabilities of the convolution/backprojection reconstruction algorithm. Increased attention is being directed toward techniques which take advantage of apriori information (e.g., measurement noise statistics, average pixel (picture element) correlations, shape of scanning beam). Such algorithms (e.g., Kalman filter) are generally characterized (and controlled) by a pixel error-covariance matrix. It is important to be able to compare these algorithms with convolution/backprojection techniques. This paper provides that capability by deriving a convenient expression for the pixel error-covariance for algorithms based on the Radon inversion formula. The expression is than adapted to a specific fan-beam data acquisition scheme. Using this expression, the convolution/backprojection algorithm is compared with two algorithms based on the Kalman filter for a variety of scanning configurations.**

I. Introduction

The discrete approximation to the Radon inversion formula is the basis for most image reconstruction algorithms for conventional CT. The popularity of this formula is due to the fact that it admits a straightforward, rapid computation of a reconstruction estimate with minimum data storage requirements. The accuracy of the technique (often referred to as convolution/backprojection) is limited by the assumptions of the Radon inversion derivation: noise-free measurements; effective continuity of the projections.

For some diagnostic applications of CT, these assumptions are less viable. Gated data collection for cardiac scanning generates discontinuous projections. It is necessary to minimize single-exposure dosage, since repeated exposure is required in dynamic scanning; but this reduces the signal-to-noise ratio.

These considerations have motivated the investigation of reconstruction techniques which are capable of more accurately modeling a specific scanning environment. One such technique utilizes a special case of the general Kalman filter which can be characterized by a pixel error-covariance matrix parameterized by

Manuscript received April 7, 1980; revised September 24, 1980. Paper recommended by A. Ephremides, Past Chairman of the Estimation Committee. This work was supported in part by the Technicare Corporation and in part by the Department of Energy under Contract DE AC01-79-ET-29363.

D. Rohler is with Case Western Reserve University, Cleveland, OH 44106, and the Technicare Corporation, Solon, OH 44139.

P. S. Krishnaprasad was with Case Western Reserve University, Cleveland, OH 44106. He is now with the University of Maryland, College Park, MD 20742.

0018-9286/81/0400-0483$00.75 ©1981 IEEE

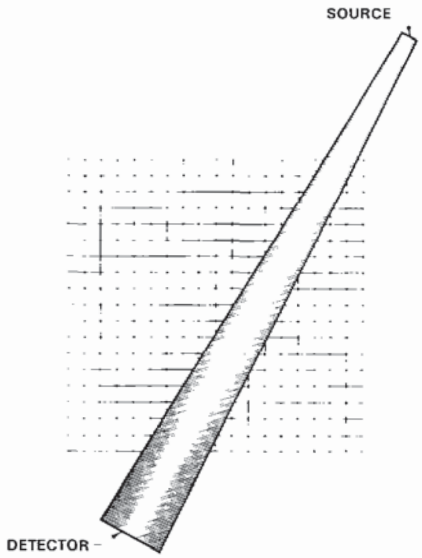

Fig. 1. Variable pixel contribution for the ith measurement defined by Ai.

1) the geometry of the data collection,
2) the statistics of the image model, and
3) the measurement statistics.

In this paper we derive a recursive expression for the pixel error covariance matrix which is applicable to algorithms of the nature of the Radon inversion formula. This expression provides the capability of a direct comparison of the convolution/backprojection algorithm with the Kalman filter algorithm for various combinations of the above parameters.

The expression is first derived in the general case (Sections III and IV) and then, for the sake of illustration, is adapted to a specific, rapid data acquisition system using a specific realization of the Radon inversion formula (Section V).

In this context, we make comparisons (Section VII) of the expected error for three reconstruction techniques:

1) convolution/backprojection,
2) (optimal) Kalman filter (Section VI), and
3) suboptimal (diagonal covariance) Kalman filter (Section VI) for some representative combinations of parameters.

Based on the results of these comparisons, we make some observations about the relative accuracy of these techniques as compared with the relative computational and storage requirements.

II. Model of the Scanning Process

In this paper, we assume a rectangular array of pixels partitioning the object to be reconstructed. We then arrange this pixel array in a column vector. We also assume that the detector measurements are conveniently organized into a linear sequence. We need the following definitions.

1) x: column vector representing the pixel attenuation values which will be estimated by the reconstruction;
2) y_i: scalar representing the ith measurement;
3) A_i: row vector of measurement coefficients in which each element represents the contribution of one pixel to the ith measurement;
4) w_i: scalar representing the zero-mean error component of the ith measurement.

We then have a linear system which approximates the scanning process

$$y_i = A_i x - w_i. \tag{1}$$

This model is adequate to include the representation of a realistic finite-width measurement beam with an arbitrary distribution as illustrated in Fig. 1. By suitably defining the vector, A_i, for each i, any scanning geometry may be modeled.

III. Representation of the (Radon Inversion) Reconstruction Process

A discrete approximation to the Radon inversion formula produces an estimate, $\hat{x}$, of the desired pixel attenuation values, x. The implementation of estimates of this type (e.g., the convolution/backprojection technique) is generally organized for rapid computation. The estimate may be presented in the form

$$\hat{x} = \Sigma_i K_i y_i \tag{2}$$

where each element of column vector K_i represents the contribution of the ith measurement to the estimate of one pixel value. Section IV illustrates the derivation of the K_i vectors for a typical fan-beam reconstruction algorithm. To derive a recursive form of the pixel error-covariance, we define the following intermediate pixel estimates, $\hat{x}_i$, using the first i measurements

$$\hat{x}_i = \sum_{j=1}^{i} K_j y_j$$
$$\hat{x}_o = 0. \tag{3}$$

If we define

$$\Lambda_i = [K_1 \ K_2 \ K_3 \ \cdots \ K_i],$$
$$\Omega_i' = [A_1 \ A_2 \ A_3 \cdots \ A_i],$$
$$Y_i' = [y_1 \ y_2 \ y_3 \ \cdots \ y_i], \text{ and}$$
$$W_i' = [w_1 \ w_2 \ w_3 \ \cdots \ w_i], \tag{4}$$

then (3) may be conveniently written in matrix notation as

$$\hat{x}_i = \Lambda_i Y_i. \tag{5}$$

Combining (1), (4), and (5), we may write the error, $\tilde{x}_i$, in the ith estimate as

$$\tilde{x}_i = \hat{x}_i - x$$
$$= \Lambda_i[\Omega_i x + W_i] - x$$
$$= [\Lambda_i \Omega_i - I]x + \Lambda_i W_i$$
$$= x_{i-1} + K_i A_i x + K_i w_i. \tag{6}$$

IV. Error-Covariance for the (Radon Inversion) Reconstruction Process

If we assume that x is not correlated with any w_j and we assume that w_j and w_k are uncorrelated when $k \neq j$, and we define P_o and R_i by

$$P_o = E(\hat{x}_o - x)(\hat{x}_o - x)' = Exx'$$
$$R_i = Ew_i'w_i, \tag{7}$$

then the pixel error-covariance, P_i, after applying the first i measurements may be written from (6) as

$$P_i = E\tilde{x}_i\tilde{x}_i'$$
$$= P_{i-1} + K_i A_i P_o(\Lambda_{i-1}\Omega_{i-1} - I)' + (\Lambda_{i-1}\Omega_{i-1} - I)P_o A_i' K_i' + K_i R_i K_i'. \tag{8}$$

We thus have in (8) a recursive expression for P_i which is dependent only on the following system parameters.

1) P_o is the initial pixel error-covariance matrix representing the class of objects to be reconstructed. If this parameter matrix is set to the identity matrix, I, the resultant P_i will represent a 'worst-case' analysis. If, however, P_i is to represent an error estimate which is numerically accurate, a reasonable attempt should be made to characterize the pixel relationship of the class of objects to be reconstructed.

2) A_i is the scanning operator for measurement i. The row vectors, A_i, allow the details of the scanning beam geometry to be factored into

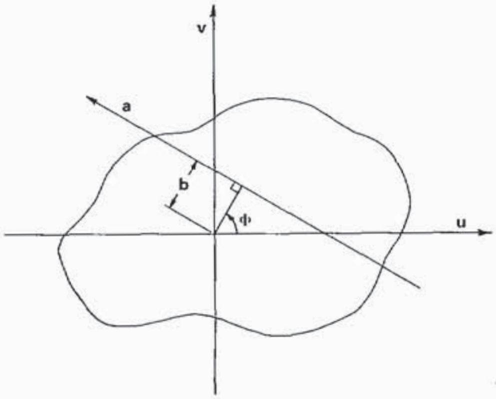

Fig. 2. Illustration of Radon transform coordinates.

the analysis, thus providing the means for determining the suitability of a reconstruction algorithm for a given scanning technique.

3) K_i is the vector gains in a general reconstruction algorithm.

4) R_i is the noise variance of the ith measurement.

V. Derivation of the K_i Vectors for a Typical Reconstruction Algorithm

The Radon inversion process is based on a continuous model of the scanning process. A density integral or line integral, $y(b,\Phi)$ is defined for $B\in(-\infty,\infty)$, $\Phi\in(0,2\pi)$ as the integral of a density function along a line perpendicular to a radial line inclined at an angle Φ at a distance b from the origin (Fig. 2):

$$y(b,\Phi)=\int_{-\infty}^{\infty} x(b\cos\Phi-a\sin\Phi,\, b\sin\Phi+a\cos\Phi)\,da. \tag{9}$$

This functional transformation mapping density functions defined on a plane into line integral functions defined on the surface of a cylinder, is often designated as the Radon transform.

In his famous paper of 1917 [4], Radon derives an inversion formula for this functional transformation

$$x(u,v)=\frac{1}{4\pi^2}\int_o^{2\pi}\int_{-\infty}^{\infty}\left(-\frac{1}{c}\right)\frac{\partial}{\partial b}y(b,\Phi)\,db\,d\Phi \tag{10}$$

where

$$c=b-u\cos\Phi-v\sin\Phi.$$

This Radon inversion formula is valid for suitable regularity conditions on the density functions.

There are a number of references [1], [2] which present discrete approximations to the Radon inversion formula (10). These approximations may include a coordinate transformation replacing Radon's (b,Φ) coordinates with coordinates corresponding to some data acquisition geometry.

We will illustrate our result with a data acquisition configuration consisting of N fan projections whose focal points are uniformly spaced $(\Delta\theta=2\pi/N)$ on a ring of radius D (Fig. 3). Each fan projection consists of M measurement rays emanating from the focal point and spaced so that they intercept the line perpendicular to the focus-origin line in equal linear increments, Δs (Fig. 4).

A discrete approximation to the Radon inversion formula in this (s,θ) coordinate system is given by Horn [2]. We start with a result equivalent to that presented by Horn but with some notational differences.

Let d index the fan projections $(d=1,\cdots,N)$ and let $\theta(d)$ represent the angle of the focus-origin line for projection d. Let t index the rays within a projection $(t=1,\cdots,M)$ and let $S(d,t)$ represent the projection line coordinate for ray t. Let $y(d,t)$ represent the value of the measurement for ray t of projection d.

Then this discrete approximation to the Radon inversion formula will

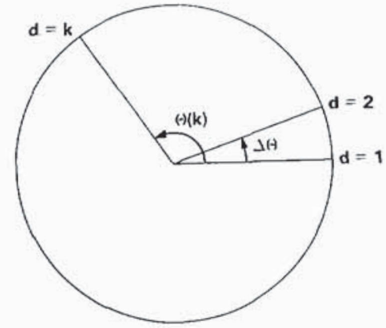

Fig. 3. Location of fan projection focal points.

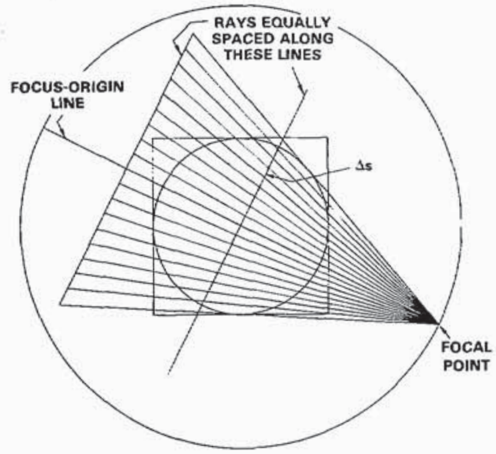

Fig. 4. Configuration of one fan projection.

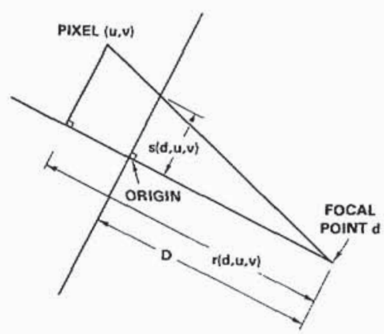

Fig. 5. Backprojection geometry for discrete Radon inversion formula.

produce an attenuation estimate for a pixel at coordinates (u,v) given by the backprojection expression (Fig. 5)

$$\begin{aligned}\hat{x}(u,v)&=\frac{1}{4\pi^2}\sum_{d=1}^{N}G[d,s(d,u,v)]\frac{D^2}{r^2(d,u,v)}\Delta\theta\\ r(d,u,v)&=D-[u\cos\theta(d)+v\sin\theta(d)]\\ s(d,u,v)&=D[u\sin\theta(d)-v\cos\theta(d)]/r(d,u,v)\end{aligned} \tag{11}$$

and $G(d,s)$ is obtained by interpolation from the available evaluations, $Z(d,\tau)$, of the convolutional sums with an appropriate interpolating function $q(\ )$

$$G(d,s)=\sum_{\tau=1}^{M}q(s-S(d,\tau))Z(d,\tau)\Delta s. \tag{12}$$

The convolution results, $Z(d,\tau)$, are computed using an appropriate convolution kernel, $F(\ )$, and a measurement premultiplication

$$Z(d,\tau)=\sum_{t=1}^{M}F(\tau-t)\frac{D}{\sqrt{D^2+S^2(d,t)}}y(d,t)\Delta s. \qquad (13)$$

Reasonable choices for q and F are

$$q(s)=\begin{cases}1-\dfrac{|s|}{\Delta s} & |s|\leq\Delta s\\ 0 & |s|>\Delta s\end{cases} \qquad (14)$$

$$F(\tau)=\begin{cases}-2/(\tau\Delta s)^2 & \tau \text{ odd}\\ 0 & \tau\neq 0,\ \tau \text{ even}\\ \pi^2/2(\Delta s)^2 & \tau=0.\end{cases} \qquad (15)$$

We may consolidate the above expressions (11)–(13),

$$\hat{x}(u,v)=\frac{1}{4\pi^2}\sum_{d=1}^{N}\frac{D^2\Delta\theta}{r^2(d,u,v)}\cdot\sum_{\tau=1}^{M}q[s(d,u,v)-S(d,\tau)]\Delta s\sum_{t=1}^{M}F(\tau-t)\cdot\frac{D}{\sqrt{D^2+S^2(d,t)}}y(d,t)\Delta s \qquad (16)$$

in order to extract the desired K vectors.

$$K_{d,t}(u,v)=\frac{D^3(\Delta s)^2\Delta\theta}{4\pi^2r^2(d,u,v)\sqrt{D^2+S^2(d,t)}}\cdot\sum_{\tau=1}^{M}q(s(d,u,v)-S(d,\tau))F(\tau-t) \qquad (17)$$

$$\hat{x}(u,v)=\sum_{d=1}^{N}\sum_{t=1}^{M}K_{d,t}(u,v)y(d,t). \qquad (18)$$

It is clear that by suitable reindexing, one can convert (18) into the form (2).

The result of this section (17) and (18), combined with (8) of Section IV, provides a powerful tool for characterizing the accuracy of reconstruction algorithms derived from the Radon inversion formula, and making comparisons with algorithms based on the Kalman filter. The next section reviews two algorithms based on the Kalman filter and Section VII presents some sample comparisons.

VI. Algorithms Based on the Kalman Filter

It has been suggested [3] that a special case of the general Kalman filter may be applied to the reconstruction problem. This technique has the advantage of utilizing apriori values of the initial pixel error-covariance, the geometric measurement operators and the measurement noise to obtain an optimal estimate of the pixel attenuation values.

For completeness, we include the details of the Kalman reconstruction as presented in [3].

$$\hat{x}_i=\hat{x}_{i-1}+K_i(y_i-A_i\hat{x}_{i-1}) \qquad (19)$$

$$K_i=P_iA_i'(R_i^e)^{-1} \qquad (20)$$

$$R_i^e=A_iP_iA_i'+R_i \qquad (21)$$

$$P_{i+1}=P_i-K_iR_i^eK_i' \qquad (22)$$

$$\hat{x}_o=Ex \qquad (23)$$

$$P_o=E(\hat{x}_o-x)(\hat{x}_o-x)'. \qquad (24)$$

It has also been suggested [3] that a simplification of the Kalman filter may be used for reconstruction. This simplification involves the elimination of all but the diagonal terms of the pixel error-covariance matrix, thus the algorithm cannot make use of pixel cross-covariance information. This technique retains the advantage of utilizing the apriori pixel variance, the geometric measurement operators and the measurement noise to obtain a suboptimal estimate of the pixel attenuation values.

Wood, *et al.* [3] present the details of this algorithm and demonstrate its convergence. For completeness, we include the details of the diagonal Kalman reconstruction:

$$\hat{x}_i=\hat{x}_{i-1}+K_i(y_i-A_i\hat{x}_{i-1}) \qquad (19)$$

$$K_i=r_i^{-1}D_iA_i' \qquad (25)$$

$$r_i=(A_iD_iA_i'+R_i)/\lambda_i \qquad (26)$$

$$D_{i+1}=\mathrm{diag}[(I-K_iA_i)D_i(I-K_iA_i)'+K_iR_iK_i'] \qquad (27)$$

$$P_{i+1}=P_i-r_i^{-1}[D_iA_i'A_iP_i+P_iA_i'A_iD_i]+r_i^{-2}[D_iA_i'+r_i^eA_iD_i] \qquad (28)$$

$$r_i^e=A_iP_iA_i'+R_i \qquad (29)$$

$$\hat{x}_o=Ex \qquad (23)$$

$$P_o=E(\hat{x}_o-x)(\hat{x}_o-x)' \qquad (24)$$

where $T=\mathrm{diag}(Q)$ is defined by

$$T_{ij}=\begin{cases}Q_{ij} & i=j\\ 0 & i\neq j\end{cases}$$

and λ_i is an adjustment factor $0<\lambda_i<2$.

TABLE I

Fig. 6	Fig. 7	Fig. 8	Fig. 9
$N=15$	$N=15$	$N=15$	$N=15$
$M=9$	$M=17$	$M=17$	$M=9$
$W=0.5$ cm	$W=0.5$ cm	$W=0.5$ cm	$W=0.5$ cm
$\alpha=1.0$	$\alpha=1.0$	$\alpha=1.0$	$\alpha=1.0$
$\sigma=0.01$	$\sigma=0.01$	$\sigma=2.0$	$\sigma=2.0$
$R=0.0004$	$R=0.0004$	$R=0.0004$	$R=0.0004$
$\lambda=1.0$	$\lambda=1.0$	$\lambda=1.0$	$\lambda=1.0$

VII. Algorithm Comparisons

It is now possible to make direct comparisons of these algorithms by plotting $\sqrt{\mathrm{tr}\,P_i}$ as a function of i for the pixel error-covariance expressions corresponding to each algorithm:

1) convolution/backprojection (8), (17)
2) Kalman filter (20)–(22)
3) diagonal Kalman filter (25)–(28).

We make these comparisons for four different combinations of system parameters. For each of these combinations, the geometric configuration of Figs. 3–5 may be used as a model. The M samples for each of N fan projections are arranged such that one sample goes through the origin and the remaining samples span the pixel matrix symmetrically. Each sample is modeled as a uniformly distributed rectangular beam of width, W. The initial pixel error-covariance is modeled with a Gaussian distribution parameterized by α and σ.

$$P_o[(u_1,v_1),(u_2,v_2)]=\alpha e^{-[(u_1-u_2)^2+(v_1-v_2)^2]/2\sigma^2}. \qquad (30)$$

The measurement noise statistics in these examples is assumed constant for all measurements. The measurement noise variance is defined by the parameter R. The pixel error-covariance values are computed for a 7×7 matrix of 1 centimeter pixels.

Each of the attached Figs. 6–9 contains a graph for each algorithm showing the decrease of the expected pixel *RMSD* as each fan of measurements is applied. Table I defines the combination of parameters illustrated by each of the figures.

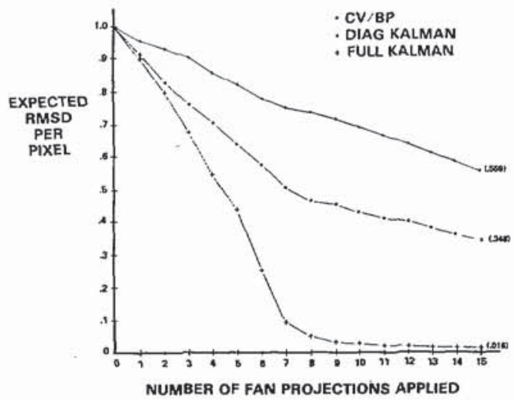

Fig. 6.

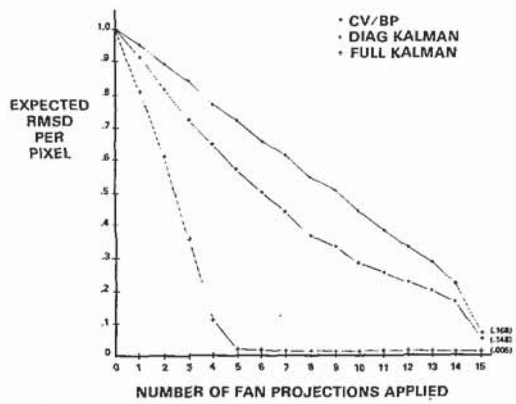

Fig. 7.

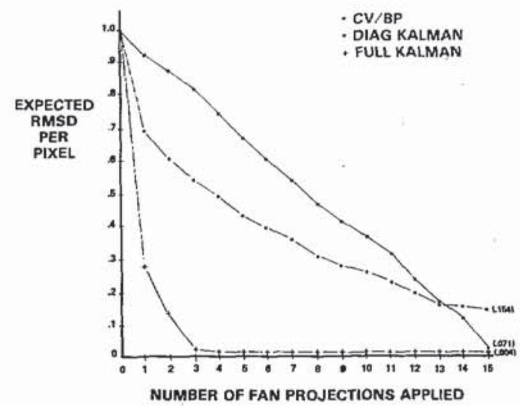

Fig. 8.

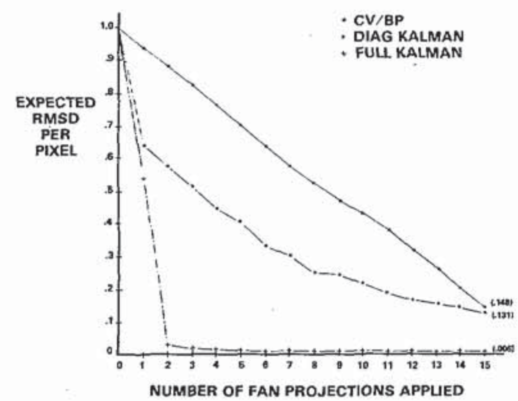

Fig. 9.

VIII. Discussion

We have presented a technique for comparing Radon inversion and Kalman reconstructions. The technique is illustrated using four sample configurations. In each of the examples the full Kalman filter was superior. The diagonal Kalman filter seems to perform better than the convolution/backprojection technique when the measurement data are limited. The convolution/backprojection reconstruction is significantly improved when the number of measurements per projection is increased (Fig. 7 versus Fig. 6). This increase corresponds to a more accurate approximation of the Radon inversion formula. The reconstruction is also improved significantly when the object pixel correlation function, P_o, is broadened (Fig. 9 versus Fig. 6). This improvement comes about because the broader correlation function more closely matches the regularity conditions of the Radon inversion formula.

The measurement noise is another important parameter in the analysis of reconstruction algorithms. Because the variance of the measurement noise is inversely proportional to the number of detected photons, it may be desirable to make use of a noise term that varies with measurement. This concept can be used in expected error computations only if the class of objects is well defined. However, the algorithms derived from the Kalman filter can incorporate this feature dynamically.

It is important to note the storage and computational requirements of each algorithm. The following table summarizes the approximate requirements for reconstruction of a $Q\times Q$ pixel matrix using N projections with M rays/projection. The expressions for the Kalman based algorithms do not include the computation of the A_i vectors.

	Storage	Computations
CV/BP	Q^2	$(10Q^2+2M^2+M)N$
Full Kalman	Q^4+3Q^2	$(5Q^4+7Q^2)MN$
Diagonal Kalman	$4Q^2$	$16QMN.$

In this exercise we have not used any of the fast implementations of the Kalman filter [5].

We feel that this technique for computing the pixel error-covariance for Radon inversion reconstructions has significant potential for characterization of specific implementations of the Radon inversion formula. The advantage of the pixel error-covariance characterization lies in the ability to accurately model the scanning process. The primary disadvantage is the enormous data storage and computational requirements. To analyze the reconstruction of a $Q\times Q$ matrix using N projections with M rays/projection, (8) requires $2Q^4$ storage locations and approximately $13Q^2MN$ computations. This disadvantage may be minimized if it can be shown that a smaller subset of pixels can be used to obtain comparisons similar to those obtained with a full set of pixels.

It may also be possible to redefine portions of the state space into pixel regions so that the global effect of these regions can be factored into the algorithm while the value of Q is kept to a minimum.

There is a significant difference in accuracy and computation and storage requirements between the diagonal Kalman filter and the full Kalman filter. It may be desirable to select an algorithm which lies somewhere between these two extremes. We have at hand a family of such suboptimal algorithms which are parameterized by the amount of available storage. Details will appear elsewhere [6].

References

[1] A. V. Lakshminarayanan, "Reconstruction from divergent ray data," Dep. Comput. Sci., State Univ. New York, Buffalo, Tech. Rep. TR-92, 1975.
[2] B. K. P. Horn, "Fan-beam reconstruction methods," *Proc. IEEE*, vol. 67, Dec. 1979.
[3] S. L. Wood, M. Martin and A. Macovski, "Stochastic methods applied to medical image reconstruction," in *Proc. IEEE Conf. Decision Contr.*, Dec. 1977.
[4] Johann Radon, "On the determination of functions by their integral values along certain manifolds," *Ber. Saechsibbe Akad. Wiss.*, vol. 69, pp. 262–278, 1917, translated by Analogic Corp., 1976.
[5] T. Kailath,, "A view of three decades of linear filtering," *IEEE Trans. Inform. Theory*, vol. IT-20, pp. 146–181, Mar. 1974.
[6] D. Rohler, Ph.D. dissertation, Case Western Reserve Univ., Cleveland, OH, 1981 (to appear).