

Motion camouflage with sensorimotor delay

P. V. Reddy, E. W. Justh, and P. S. Krishnaprasad

Abstract—In recent work, a particular high-gain feedback law was shown to drive a pursuer-evader system arbitrarily close to a state of motion camouflage in finite time. However, data collected from bat-insect encounters, in which a strategy akin to motion camouflage is used by the bat to pursue the insect, reveal that a modest feedback gain is used, and significant sensorimotor delay is present. Therefore, we revisit our earlier analysis (in CDC 2006), but with sensorimotor delay incorporated into the model. We derive constraints among parameters such as feedback gain, sensorimotor delay, and relative speed, for which it is possible to guarantee performance of the feedback law in achieving motion camouflage. Besides helping us to better understand pursuit in nature, such pursuit laws have applications in missile guidance and unmanned vehicle control.

I. INTRODUCTION

The term “motion camouflage” originated from the study of stealthy pursuit behaviors in visual insects - in particular, hoverflies [10] and later dragonflies [6]. The distinguishing characteristic of motion camouflaged pursuit is that the pursuer moves in such a way as to minimize the visual cues available to the evader to detect that the pursuer is closing in. With compound eyes, insects are sensitive to optic flow, and hence motion parallax, but are less sensitive to looming (i.e., decrease in absolute distance). There are thus two distinct motion camouflage strategies: motion camouflage with respect to a finite point, and motion camouflage with respect to a point at infinity. In the former, a fixed object (either between the pursuer and evader or behind the pursuer, as seen from the evader) serves to camouflage the pursuer. By remaining along the line joining the object and evader, the evader cannot distinguish the motion of the pursuer from the motion parallax due to the fixed object, and so does not detect the approach of the pursuer. In the other type, motion camouflage with respect to a point at infinity, the pursuer approaches in such a way as to appear to the evader to lie at a constant bearing. The evader perceives the pursuer as

having the same motion parallax as a point at infinity, and hence does not perceive it as a threat.

In this paper, we henceforth only consider motion camouflage with respect to a point at infinity, and use the term “motion camouflage” to refer only to motion camouflage with respect to a point at infinity. It is this type of motion camouflage for which a simple feedback law (closely connected to the PPNG law in missile guidance [9]) has been analyzed (in the absence of delays) [5], [8].

In addition to insect encounters, it has also been observed that an echolocating bat uses a strategy which is geometrically indistinguishable from motion camouflage to intercept prey insects [2]. Using the term “motion camouflage” in the context of the bat-insect engagement is somewhat misleading, because the engagements take place in the dark, and echolocation rather than vision is the sensory mechanism used by the pursuer. There is thus no reason to believe that the bat adopts this strategy for purposes of camouflage. It is more plausible to suspect that this strategy enables the bat to make the most effective use of its sensory processing apparatus, so that the sensory-motor feedback system which drives the bat toward the target insect will perform well. (Better understanding of sensory-motor control in the bat is a main goal of the Auditory Neuroethology Lab directed by Prof. Cynthia Moss at the University of Maryland; see <http://www.bsos.umd.edu/psyc/batlab/>.)

The geometry of motion camouflage can be described independently of any particular feedback law which gives rise to it, as in, e.g., [3]. However, it is important and useful to distinguish between a *state* of motion camouflage and the feedback control law which *drives* the system toward a state of motion camouflage.

A biologically plausible feedback law to drive a system into a state of motion camouflage was first presented in [5]. The only free parameter in this feedback law is a scalar gain. In the absence of delays (and noise) in the sensory-motor system, it has been shown that the higher the gain, the greater is the effectiveness of the feedback law in driving the pursuer-evader system into a state of motion camouflage [5], [8]. However, in practice there will always be sensorimotor delay which will limit the value of gain that can be used. Using experimental data, the values of gain and delay can be estimated for bat-insect engagements [2], [7]. Here we investigate the effects of delay to learn what values of gain are suitable, and characterize how “close” to a state of motion camouflage the system can be driven.

There is a natural generalization of motion camouflage engagements to three dimensions [8], but to focus on the effects of delay, we restrict to the planar setting. For an

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expanded investigation of delay and feedback gain in motion camouflage pursuit, see [7].

This paper is organized as follows. In Section II, the motion camouflage model from [5] is summarized. In Section III, a sensorimotor delay (or equivalently, a feedback delay) is introduced, and the system with delay is analyzed along the lines of the analysis in [5] for the delay-free setting. In Section IV, an interpretation of the analytical results is given, and in Section V, a numerical example is presented for further illustration. Finally, conclusions and directions for future work are contained in Section VI.

II. MOTION CAMOUFLAGE MODEL

The pursuer and evader are modeled as point particles which move at constant speed in the plane subject to steering control. Generalization to three-dimensional motion and arbitrary positive speeds is described in [8]. However, to focus on the effects of delay, we restrict attention to the simpler setting of planar motion with constant speeds, and note that the generalization to three-dimensional motion and non-constant speeds is straightforward. For zero delay, the analysis here reduces precisely to the one presented in [5].

A. Trajectory and frame evolution

Particles moving at constant speed subject to continuous steering controls trace out trajectories which are C^2 , i.e., which are twice continuously differentiable. Without loss of generality, we may assume that the pursuer particle moves at unit speed, and the evader particle moves at speed $\nu > 0$ (i.e., ν corresponds to the ratio of speeds of the pursuer and evader).

The motion of the pursuer is described by

$$\begin{aligned}\dot{\mathbf{r}}_p &= \mathbf{x}_p, \\ \dot{x}_p &= y_p u_p, \\ \dot{y}_p &= -x_p u_p,\end{aligned}\quad (1)$$

and the motion of the evader is described by

$$\begin{aligned}\dot{\mathbf{r}}_e &= \nu \mathbf{x}_e, \\ \dot{x}_e &= \nu y_e u_e, \\ \dot{y}_e &= -\nu x_e u_e,\end{aligned}\quad (2)$$

where the steering control of the evader, u_e , is prescribed, and the steering control of the pursuer, u_p , is given by a feedback law. The orthonormal frame $\{\mathbf{x}_p, \mathbf{y}_p\}$, which is the planar natural Frenet frame for the pursuer particle, evolves with time as the pursuer particle moves along its trajectory (described by \mathbf{r}_p). Similarly, $\{\mathbf{x}_e, \mathbf{y}_e\}$ is the planar natural Frenet frame corresponding to the evader particle [1], [4].

Here we assume $\nu < 1$, so that the pursuer moves faster than the evader. Figure 1 illustrates (1) and (2).

B. Feedback law for motion camouflage

We define

$$\mathbf{r} = \mathbf{r}_p - \mathbf{r}_e, \quad (3)$$

so that \mathbf{r} is the relative position of the pursuer with respect to the evader, and we refer to \mathbf{r} as the ‘‘baseline vector.’’

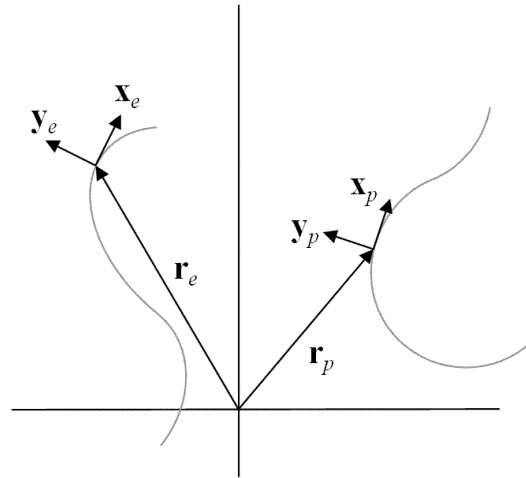


Fig. 1. Illustration of the trajectories and natural Frenet frames for the planar pursuer-evader engagement. (Figure from [5].)

We define the notation \mathbf{q}^\perp to represent the vector \mathbf{q} rotated counter-clockwise in the plane by an angle $\pi/2$ [5], [8]. When there is no delay associated with incorporating sensory information into the feedback law, we let

$$u_p = -\mu \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^\perp \right), \quad (4)$$

where $\mu > 0$ is a gain parameter [5], [8]. However, if there is a delay τ in the incorporation of sensory information, then we substitute $u_p(t - \tau)$ for u_p in equation (1).

Observe that (4) is only well defined for $|\mathbf{r}| \neq 0$. For this feedback law, it is assumed that $|\mathbf{r}| \neq 0$ initially, and the engagement is analyzed (for finite time) until $|\mathbf{r}|$ reaches a value $r_0 > 0$ [5], [8].

C. Distance from motion camouflage

The function

$$\Gamma = \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} \right) \quad (5)$$

describes how far the pursuer-evader system is from a state of motion camouflage [5], [8]. The system is in a state of motion camouflage when $\Gamma^2 = 1$, with $\Gamma = -1$ corresponding to pure shortening of the baseline vector, and $\Gamma = +1$ corresponding to pure lengthening. The difference $\Gamma - (-1) > 0$ is a measure of the distance of the pursuer-evader system from a state of motion camouflage (with baseline shortening).

For (5) to be well defined, we must have $|\mathbf{r}| > 0$ (as discussed in the previous subsection), as well as $|\dot{\mathbf{r}}| > 0$. However, the latter condition is ensured by the assumption that $0 < \nu < 1$, since $|\dot{\mathbf{r}}| \geq 1 - \nu$.

Differentiating Γ along trajectories of (1) and (2) gives

$$\begin{aligned}\dot{\Gamma} &= \left(\frac{\dot{\mathbf{r}} \cdot \dot{\mathbf{r}} + \mathbf{r} \cdot \ddot{\mathbf{r}}}{|\mathbf{r}| |\dot{\mathbf{r}}|} \right) - \left(\frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{|\mathbf{r}|} \right) \left(\frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{|\mathbf{r}|^3} \right) - \left(\frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{|\mathbf{r}|} \right) \left(\frac{\dot{\mathbf{r}} \cdot \ddot{\mathbf{r}}}{|\dot{\mathbf{r}}|^3} \right) \\ &= \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|} (1 - \Gamma^2) + \frac{1}{|\dot{\mathbf{r}}|} \left[\frac{\mathbf{r}}{|\mathbf{r}|} - \left(\frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{|\mathbf{r}|} \right) \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} \right] \cdot \ddot{\mathbf{r}}.\end{aligned}\quad (6)$$

Defining w by

$$w = - \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^\perp \right), \quad (7)$$

so that $u_p = \mu w$, and noting that

$$\begin{aligned} \left[\frac{\mathbf{r}}{|\mathbf{r}|} - \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} \right) \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} \right] &= \left[\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \left(\frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} \right)^\perp \right] \left(\frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} \right)^\perp \\ &= -\frac{w}{|\dot{\mathbf{r}}|^2} \dot{\mathbf{r}}^\perp, \end{aligned} \quad (8)$$

we see that (6) becomes

$$\dot{\Gamma} = \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|} (1 - \Gamma^2) - \frac{1}{|\dot{\mathbf{r}}|} \left[\frac{w}{|\dot{\mathbf{r}}|^2} \dot{\mathbf{r}}^\perp \right] \cdot \ddot{\mathbf{r}}. \quad (9)$$

Furthermore, $\dot{\mathbf{r}}^\perp = \mathbf{y}_p - \nu \mathbf{y}_e$, and $\ddot{\mathbf{r}} = \mathbf{y}_p u_p(t - \tau) - \nu^2 \mathbf{y}_e u_e$, so that

$$\dot{\mathbf{r}}^\perp \cdot \ddot{\mathbf{r}} = [1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)] u_p(t - \tau) + \nu^2 [\nu - (\mathbf{x}_p \cdot \mathbf{x}_e)] u_e. \quad (10)$$

Thus,

$$\begin{aligned} \dot{\Gamma} &= \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|} (1 - \Gamma^2) - \frac{1}{|\dot{\mathbf{r}}|} \left(\frac{w}{|\dot{\mathbf{r}}|^2} \right) [1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)] u_p(t - \tau) \\ &\quad - \frac{1}{|\dot{\mathbf{r}}|} \left(\frac{w}{|\dot{\mathbf{r}}|^2} \right) \nu^2 [\nu - (\mathbf{x}_p \cdot \mathbf{x}_e)] u_e \\ &\leq \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|} (1 - \Gamma^2) - \frac{1}{|\dot{\mathbf{r}}|} \left(\frac{w}{|\dot{\mathbf{r}}|^2} \right) [1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)] u_p(t - \tau) \\ &\quad + \frac{|w|}{|\dot{\mathbf{r}}|} \left[\frac{\nu^2(1 + \nu)}{(1 - \nu)^2} \right] u_{max}. \end{aligned} \quad (11)$$

where $u_{max} = \max |u_e|$. We would like to understand what happens to the term involving $u_p(t - \tau)$ above, since this is where the effects of sensorimotor delays enter our model.

III. DELAY ANALYSIS

In particular, we decompose $u_p(t - \tau)$ into two terms:

$$u_p(t - \tau) = u_p(t) + [u_p(t - \tau) - u_p(t)]. \quad (12)$$

We know from the analysis for delay $\tau = 0$ that the first term on the right-hand-side of (12) will help us to achieve $\dot{\Gamma} \leq 0$ (under certain conditions). The second term in (12) is something we would like to bound.

By the Mean Value Theorem, since u_p is continuous, there exists $t^* \in (t - \tau, t)$ such that

$$u_p(t - \tau) - u_p(t) = -\tau \dot{u}_p(t^*) = -\tau \mu \dot{w}(t^*). \quad (13)$$

Also,

$$\begin{aligned} \dot{w} &= - \left[\frac{d}{dt} \left(\frac{\mathbf{r}}{|\mathbf{r}|} \right) \cdot \dot{\mathbf{r}}^\perp + \left(\frac{\mathbf{r}}{|\mathbf{r}|} \right) \cdot (\ddot{\mathbf{r}}^\perp) \right] \\ &= - \left[\frac{1}{|\mathbf{r}|} \left(\dot{\mathbf{r}} - \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}} \right) \frac{\mathbf{r}}{|\mathbf{r}|} \right) \cdot \dot{\mathbf{r}}^\perp + \left(\frac{\mathbf{r}}{|\mathbf{r}|} \right) \cdot (\ddot{\mathbf{r}}^\perp) \right] \\ &= \frac{1}{|\mathbf{r}|} \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}} \right) \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^\perp \right) - \left(\frac{\mathbf{r}}{|\mathbf{r}|} \right) \cdot (\ddot{\mathbf{r}}^\perp) \end{aligned} \quad (14)$$

Thus,

$$|\dot{w}| \leq \frac{(1 + \nu)^2}{2r_0} + \mu(1 + \nu) + \nu^2 u_{max}, \quad (15)$$

for $|\mathbf{r}| > r_0$, and therefore

$$\begin{aligned} |u_p(t - \tau) - u_p(t)| \\ \leq \tau \mu \left[\frac{(1 + \nu)^2}{2r_0} + \mu(1 + \nu) + \nu^2 u_{max} \right], \end{aligned} \quad (16)$$

for $|\mathbf{r}| > r_0$. From (11) we then obtain

$$\begin{aligned} \dot{\Gamma} &\leq \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|} (1 - \Gamma^2) - \frac{1}{|\dot{\mathbf{r}}|} \left(\frac{w}{|\dot{\mathbf{r}}|^2} \right) [1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)] u_p \\ &\quad - \frac{1}{|\dot{\mathbf{r}}|} \left(\frac{w}{|\dot{\mathbf{r}}|^2} \right) [1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)] (u_p(t - \tau) - u_p) \\ &\quad + \frac{|w|}{|\dot{\mathbf{r}}|} \left[\frac{\nu^2(1 + \nu)}{(1 - \nu)^2} \right] u_{max} \\ &\leq -(1 - \Gamma^2) \left[\frac{\mu}{|\dot{\mathbf{r}}|} [1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)] - \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|} \right] \\ &\quad + \frac{|w|}{|\dot{\mathbf{r}}|} \left[\frac{\nu^2(1 + \nu)}{(1 - \nu)^2} u_{max} \right. \\ &\quad \left. + \frac{1}{|\dot{\mathbf{r}}|^2} [1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)] |u_p(t - \tau) - u_p| \right] \\ &\leq -(1 - \Gamma^2) \left[\mu \left(\frac{1 - \nu}{1 + \nu} \right) - \frac{1 + \nu}{r_0} \right] \\ &\quad + \sqrt{1 - \Gamma^2} \left[\frac{1 + \nu}{(1 - \nu)^2} \right] \\ &\quad \times \left[\nu^2 u_{max} + \tau \mu \left(\frac{(1 + \nu)^2}{2r_0} + \mu(1 + \nu) + \nu^2 u_{max} \right) \right], \end{aligned} \quad (17)$$

for $|\mathbf{r}| \geq r_0$. Taking

$$c_0 = \mu \left(\frac{1 - \nu}{1 + \nu} \right) - \frac{1 + \nu}{r_0}, \quad (18)$$

and

$$c_1 = \frac{\nu^2(1 + \nu)u_{max}}{(1 - \nu)^2}, \quad (19)$$

(17) can be rewritten as

$$\begin{aligned} \dot{\Gamma} &\leq -(1 - \Gamma^2) c_0 + \sqrt{1 - \Gamma^2} \left[c_1 + \tau \mu \right. \\ &\quad \left. \times \left[c_1 + \left(\frac{1 + \nu}{(1 - \nu)^2} \right) \left(\frac{(1 + \nu)^2}{2r_0} + \mu(1 + \nu) \right) \right] \right], \end{aligned} \quad (20)$$

for $|\mathbf{r}| > r_0$, where we note that c_0 depends on μ . Observe that for the term $-(1 - \Gamma^2)c_0$ to be negative, we require $c_0 > 0$. Thus, we require μ to satisfy the lower bound

$$\mu > \frac{(1 + \nu)^2}{(1 - \nu)r_0}. \quad (21)$$

Following the methodology of the proof of **Proposition 3.3** in [5], suppose that we are given $0 < \epsilon \ll 1$. If

$$\begin{aligned} c_0 &> \frac{1}{\sqrt{\epsilon}} \left[c_1 + \tau \mu \right. \\ &\quad \left. \times \left[c_1 + \left(\frac{1 + \nu}{(1 - \nu)^2} \right) \left(\frac{(1 + \nu)^2}{2r_0} + \mu(1 + \nu) \right) \right] \right], \end{aligned} \quad (22)$$

then for $|\mathbf{r}| > r_0$ and $(1 - \Gamma^2) > \epsilon$, (20) implies

$$\dot{\Gamma} \leq -(1 - \Gamma^2)c_2, \quad (23)$$

where

$$c_2 = c_0 - \frac{1}{\sqrt{\epsilon}} \left[c_1 + \tau\mu \times \left[c_1 + \left(\frac{1+\nu}{(1-\nu)^2} \right) \left(\frac{(1+\nu)^2}{2r_0} + \mu(1+\nu) \right) \right] \right] > 0. \quad (24)$$

We are thus led to the following result.

Proposition: Consider the system (1) with (2) and control law (4), subject to a time delay $\tau > 0$, $0 < \epsilon \ll 1$, and the following hypotheses:

- (A1) $0 < \nu < 1$ (and ν is constant),
- (A2) u_e is continuous and $|u_e|$ is bounded,
- (A3) $\Gamma_0 = \Gamma(0) < 1 - \epsilon$, and
- (A4) $|\mathbf{r}(0)| > 0$.

Let $0 < r_0 < |\mathbf{r}(0)|$, and

$$\delta = \max \left(0, \frac{1}{2} \left(\frac{1+\nu}{|\mathbf{r}(0)| - r_0} \right) \ln \left[\left(\frac{1+\Gamma_0}{1-\Gamma_0} \right) \left(\frac{2-\epsilon}{\epsilon} \right) \right] \right). \quad (25)$$

Then there exists a finite time t_1 for which $\Gamma(t_1) \leq -1 + \epsilon$ (i.e., motion camouflage is accessible in finite time), provided there exists a value of $\mu > 0$ which satisfies (21) and

$$c_2 = c_0 - \frac{1}{\sqrt{\epsilon}} \left[c_1 + \tau\mu \times \left[c_1 + \left(\frac{1+\nu}{(1-\nu)^2} \right) \left(\frac{(1+\nu)^2}{2r_0} + \mu(1+\nu) \right) \right] \right] > \delta, \quad (26)$$

where c_0 and c_1 are given by (18) and (19), respectively.

Proof: We define

$$T = \frac{|\mathbf{r}(0)| - r_0}{1 + \nu} > 0, \quad (27)$$

and note that T is a lower bound on the interval of time over which we can guarantee that $|\mathbf{r}| > r_0$. Then (as shown in [5]), for

$$\begin{aligned} c_2 &\geq (1+\nu) \frac{\tanh^{-1} \Gamma_0 - \frac{1}{2} \ln \left(\frac{\epsilon}{2-\epsilon} \right)}{|\mathbf{r}(0)| - r_0} \\ &= (1+\nu) \frac{\frac{1}{2} \ln \left(\frac{1+\Gamma_0}{1-\Gamma_0} \right) - \frac{1}{2} \ln \left(\frac{\epsilon}{2-\epsilon} \right)}{|\mathbf{r}(0)| - r_0} \\ &= \frac{1}{2} \left(\frac{1+\nu}{|\mathbf{r}(0)| - r_0} \right) \ln \left[\left(\frac{1+\Gamma_0}{1-\Gamma_0} \right) \left(\frac{2-\epsilon}{\epsilon} \right) \right], \end{aligned} \quad (28)$$

we are guaranteed to achieve $\Gamma(t_1) \leq -1 + \epsilon$ at some finite time $t_1 \leq T$. Recalling condition (24), we note that $\Gamma(t_1) \leq -1 + \epsilon$ at some finite time $t_1 \leq T$ provided $c_2 > \delta \geq 0$. \square

Remark: By taking $|\mathbf{r}(0)|$ arbitrarily large, δ may be made arbitrarily close to zero. Intuitively, this means that a large initial separation facilitates reaching the motion camouflage state. Corresponding remarks can be made about the effects of Γ_0 and ϵ on δ , as well. \square

Remark: For $\tau = 0$, **Proposition 3.3** in [5] guarantees the existence of (a range of) values of μ which meet the requirements of the above **Proposition** for any choice of $\epsilon > 0$, $0 < \nu < 1$, $|\mathbf{r}(0)| > 0$, and $\Gamma_0 < 1$. \square .

IV. INTERPRETATION

Although the **Proposition** above gives inequalities for the gain μ which, if satisfied, ensure that the system with delay passes within a tolerance ϵ of the motion camouflage state in finite time, the physical meaning of the inequalities is not immediately transparent. To clarify the interpretation of these inequalities, it is useful to first consider the limit of large r_0 .

A. $r_0 \rightarrow \infty$ limit

Let us consider the limiting situation in the **Proposition** as r_0 becomes very large, so that terms involving $1/r_0$ may be neglected in the expressions which constrain the permissible values of the feedback gain μ . We approximate

$$c_0 \approx \mu \left(\frac{1-\nu}{1+\nu} \right), \quad \delta \approx 0, \quad (29)$$

and seek μ satisfying

$$\begin{aligned} c_2 &= c_0 - \frac{1}{\sqrt{\epsilon}} \left[c_1 + \tau\mu \left[c_1 + \left(\frac{1+\nu}{(1-\nu)^2} \right) \mu(1+\nu) \right] \right] \\ &= \mu \left(\frac{1-\nu}{1+\nu} \right) - \frac{1}{\sqrt{\epsilon}} \left[c_1 + \tau\mu \left(c_1 + \mu \left(\frac{1+\nu}{1-\nu} \right)^2 \right) \right] \\ &> 0. \end{aligned} \quad (30)$$

A necessary condition for the existence of a positive value of μ satisfying (30) is

$$\frac{\tau c_1}{\sqrt{\epsilon}} < \frac{1-\nu}{1+\nu}. \quad (31)$$

If (31) is satisfied, then the value of μ which maximizes (30) is easily shown to be

$$\mu_0 = \frac{\sqrt{\epsilon}}{2\tau} \left(\frac{1-\nu}{1+\nu} \right)^2 \left[\frac{1-\nu}{1+\nu} - \frac{\tau c_1}{\sqrt{\epsilon}} \right]. \quad (32)$$

One way to approach (32) is to constrain τ and ϵ such that $\tau/\sqrt{\epsilon} = \xi = \text{constant}$, and to insist that ξ satisfy

$$\frac{\tau}{\sqrt{\epsilon}} = \xi < \xi_0 = \frac{1}{c_1} \left(\frac{1-\nu}{1+\nu} \right). \quad (33)$$

Then (32) becomes

$$\mu_0 = \frac{c_1}{2} \left(\frac{1-\nu}{1+\nu} \right)^2 \frac{\xi_0}{\xi} \left(1 - \frac{\xi}{\xi_0} \right) \quad (34)$$

If we further choose $\xi/\xi_0 = \eta \in (0, 1/2)$, then we obtain

$$\mu_0 = \frac{c_1}{2} \left(\frac{1-\nu}{1+\nu} \right)^2 \left(\frac{1}{\eta} - 1 \right) = \frac{\nu^2 u_{max}}{2(1+\nu)} \left(\frac{1}{\eta} - 1 \right). \quad (35)$$

Thus, in order to obtain an interpretation of the constraints on μ for the limiting case of r_0 large, we performed the following steps: (1) we constrained the ratio $\tau/\sqrt{\epsilon}$ to be constant, (2) we further constrained the value of this ratio to be less than 1/2 of its maximum possible value (which

is a tradeoff between ensuring that μ_0 satisfies (30) and permitting ϵ to be as small as possible for a given τ), and (3) we obtained an expression for the gain that only involves the ratio of speeds and maneuverability of the evader.

B. Finite r_0

Following a similar procedure as above, we easily compute the maximum allowable value of μ in (24) to be

$$\begin{aligned} \mu_0 &= \frac{\sqrt{\epsilon}}{2\tau} \left(\frac{1-\nu}{1+\nu} \right)^2 \\ &\quad \times \left[\frac{1-\nu}{1+\nu} - \frac{\tau}{\sqrt{\epsilon}} \left(c_1 + \left(\frac{1+\nu}{(1-\nu)^2} \right) \frac{(1+\nu)^2}{2r_0} \right) \right], \\ &= \frac{\sqrt{\epsilon}}{2\tau} \left(\frac{1-\nu}{1+\nu} \right)^2 \left[\frac{1-\nu}{1+\nu} - \frac{\tau}{\sqrt{\epsilon}} \left(c_1 + \frac{(1+\nu)^3}{2r_0(1-\nu)^2} \right) \right], \end{aligned} \quad (36)$$

but in addition, from (21), we have

$$\mu_0 > \frac{(1+\nu)^2}{(1-\nu)r_0}. \quad (37)$$

Once again, we use $\xi = \tau/\sqrt{\epsilon}$, however in place of ξ_0 , we now have

$$\zeta_0 = \left(\frac{1-\nu}{1+\nu} \right) \left(c_1 + \frac{(1+\nu)^3}{2r_0(1-\nu)^2} \right)^{-1}, \quad (38)$$

so that

$$\mu_0 = \frac{1}{2} \left(c_1 + \frac{(1+\nu)^3}{2r_0(1-\nu)^2} \right) \left(\frac{1-\nu}{1+\nu} \right)^2 \frac{\zeta_0}{\xi} \left[1 - \frac{\xi}{\zeta_0} \right]. \quad (39)$$

If we further choose $\xi/\zeta_0 = \eta \in (0, 1/2)$, then we obtain

$$\begin{aligned} \mu_0 &= \frac{1}{2} \left(c_1 + \frac{(1+\nu)^3}{2r_0(1-\nu)^2} \right) \left(\frac{1-\nu}{1+\nu} \right)^2 \left(\frac{1}{\eta} - 1 \right) \\ &= \left[\frac{\nu^2 u_{max}}{2(1+\nu)} + \frac{1+\nu}{4r_0} \right] \left(\frac{1}{\eta} - 1 \right). \end{aligned} \quad (40)$$

Thus, the term involving r_0 is balanced against the term involving c_1 (and, in turn, the relative maneuverability of the pursuer and evader).

V. NUMERICAL EXAMPLE

Figure 2 illustrates trajectories of the pursuer-evader system (1)-(2) under control law (4) for the pursuer and a sinusoidal steering control for the evader. The straight lines between the trajectories in figure 2 connect corresponding positions of the pursuer and evader at discrete, equally spaced moments in time. When these lines are parallel to each other, $\Gamma = -1$, and the system is in a state of motion camouflage [5], [8]. At the scale of figure 2, which corresponds to essentially zero feedback delay τ and a modest value for the gain μ , we see that the system quickly approaches a state of motion camouflage in this example.

We seek to illustrate the effects of sensorimotor delay on the performance of the pursuer for the example of figure 2. The parameter values used are $\nu = 0.9$, $u_{max} = 0.25$, and $r_0 = 5$. The evader steering control u_e is sinusoidal with amplitude u_{max} . Because ν is close to one, the analytically

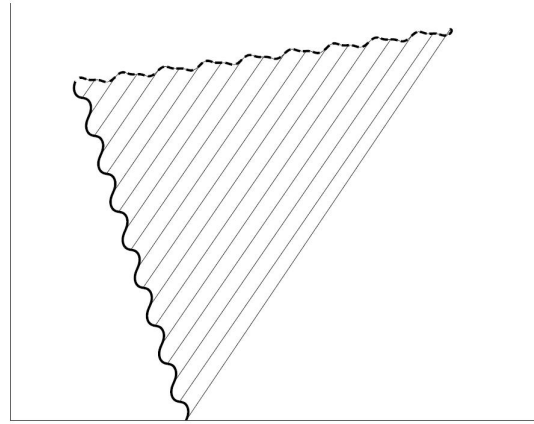


Fig. 2. Evader trajectory with sinusoidally varying steering control (dark line), and the corresponding pursuer trajectory (dashed dark line), evolving according to (1) and (2), with pursuer control given by (4).

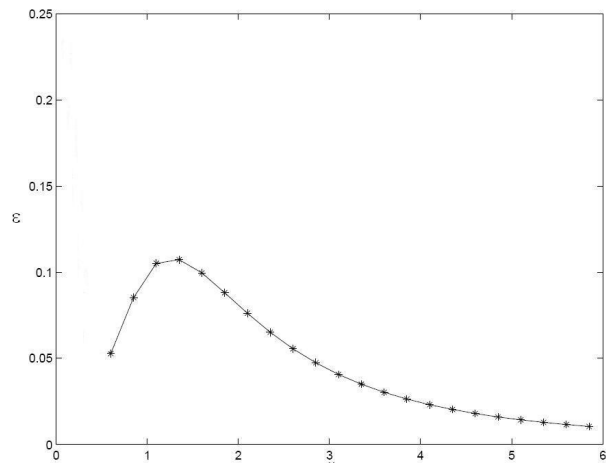


Fig. 3. Peak deviation from a motion camouflage state, ϵ , plotted as a function of increasing feedback gain μ for the delay-free case (i.e., $\tau = 0$).

derived constraints (21) and (24) are expected to be overly conservative for this example. Nevertheless, the trends identified among τ , μ , and ϵ from the analysis are expected to hold. Figures 3 and 4 confirm these trends.

For the delay-free case (i.e., $\tau = 0$), (24) becomes

$$c_2 = c_0 - \frac{c_1}{\sqrt{\epsilon}} = \mu \left(\frac{1-\nu}{1+\nu} \right) - \frac{1+\nu}{r_0} - \frac{c_1}{\sqrt{\epsilon}} > 0, \quad (41)$$

so that increasing the feedback gain μ permits smaller values of ϵ to satisfy the inequality. A higher value of μ also increases the rate at which the pursuer-evader system is driven toward a state of motion camouflage. Figure 3 shows the peak deviation of Γ from -1 (excluding an initial transient), as a function of μ , for zero feedback delay (i.e., for $\tau = 0$). In keeping with the conclusion of the **Proposition**, i.e., that there exists a finite time t_1 such that $\Gamma(t_1) \leq -1 + \epsilon$, we identify this peak deviation of Γ from -1 with ϵ . Then for μ greater than about 1.5, increasing the value of μ in figure 3 is observed to reduce ϵ .

Figure 4 shows ϵ versus μ corresponding to a feedback delay $\tau = .045$. The best performance is obtained for μ near

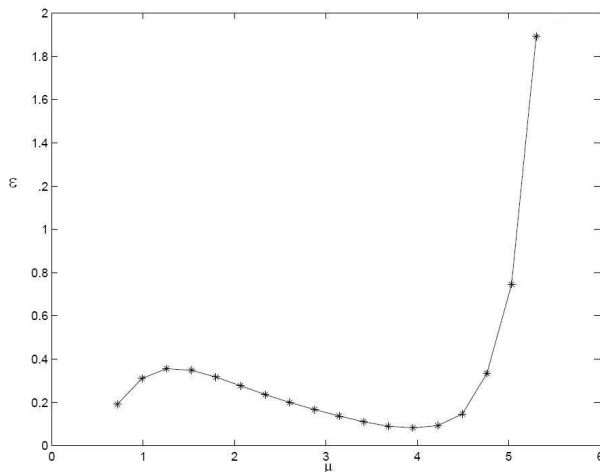


Fig. 4. Peak deviation from a motion camouflage state, ϵ , plotted as a function of increasing feedback gain μ in the presence of a sensorimotor delay $\tau = .045$.

4: for smaller values of μ , poorer performance is obtained, and for larger values of μ , the system fails to even approach a state of motion camouflage. (Recall that an upper bound on μ is reflected in (24).)

VI. CONCLUSIONS AND FUTURE WORK

There are several types of conclusions that can be reached through the approach outlined above for pursuit systems with sensorimotor or feedback delays: some apply more to biological systems, and others more to engineered systems. On the biological side, sensorimotor delay is a key performance parameter for the pursuer, as is the feedback gain μ . Although not explored here, biologically optimized values for μ are presumably influenced by sensor noise as well as by the flight performance or maneuverability of the pursuer. In control system design, we often think of gains as parameters at our disposal to optimize closed-loop performance. However, for biological motion camouflaged pursuit, it may be more compelling to think of the feedback gain as predetermined (by physical considerations) and the *delay* as the quantity “nature” tries to optimize (i.e., minimize). This could be accomplished through reflexive (rather than reflective) responses to sensory input, as well as through streamlining the sensory processing itself. In recent work [7], we have extracted the gain and delay values from data collected during bat-insect engagements as part of the process of developing an understanding of the sensorimotor design of the bat.

In our formulation of motion camouflaged pursuit, the pursuer sensor measurement (i.e., the transverse component of relative velocity) is (proportionally) fed back to control the motion of the pursuer. If the pursuer were able to *predict* future values of the transverse relative velocity, then the effects of feedback delay could be mitigated. For example, a sensor measurement of time rate of change of transverse relative velocity would be useful for this purpose. A model which keeps track of both the pursuer (center-of-mass) motion and the pursuer sensor orientation might be useful for analyzing

this additional feedback mechanism. In the echolocating bat, the head direction corresponds to the sensor orientation, and need not be aligned with the motion of the bat (although it is often observed that when the bat turns its head, there is then a tendency for the bat to bring its motion into alignment with its new head direction). Allowing for control of the sensor orientation independently from the direction of motion could be expected to simplify the task of directly sensing the rate of change of transverse relative velocity.

On the engineering side, an important practical problem is unmanned vehicle control over a wireless communication link. As pointed out in [5], [8], there is a close connection between motion camouflage and control laws used for missile guidance [9]. In this context, control over a wireless link could be used as part of the process of control system integration, or it could be used to guide the vehicle using an external sensor. In either case, satisfying the hypotheses of the **Proposition** results in a performance guarantee. If the rate of change of the transverse component of relative velocity can be sensed, then this quantity can presumably be fed back (along with the transverse relative velocity itself) to mitigate the effects of delay. If a detailed model of the target (i.e., the evader) is available, then a more target-specific approach could be used to predict target motion from the available (possibly distributed) sensor measurements. However, this deviates somewhat from the theme of simple, sensor-driven feedback laws for pursuit.

The work presented here is an initial foray into analyzing steering (or curvature) based control systems with feedback delays. By exploiting the specific structure of models based on trajectories and natural Frenet frames, we hope to derive results for other classes of nonlinear systems with delay. We then seek to learn how such results can inform our understanding of biological data, as well as provide guidance for the design of control systems for (teams of) unmanned vehicles.

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