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Motion Control and Coupled Oscillators

by P.S. Krishnaprasad

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P. S. Krishnaprasad
Department of Electrical Engineering &
Institute for Systems Research
University of Maryland
College Park, MD 20742

Abstract

It is remarkable that despite the presence of large numbers of degrees of freedom, motion control problems are effectively solved in biological systems. While feedback, regulation, and tracking have served us well in engineering, as useful solution paradigms for a wide variety of control problems including motion control, it appears that nature gives prominent roles to planning and co-ordination as well. There is also complex interplay between sensory feedback and motion planning to achieve effective operation in uncertain environments, for example, in movement on uneven terrain cluttered with obstacles. Recent investigations by neurophysiologists have brought to increasing prominence the idea of central pattern generators, – a class of coupled oscillators –, as sources of motion "scripts" as well as a means for coordinating multiple degrees of freedom. The role of coupled oscillators in motion control systems is currently under intense investigation. In this paper we examine some unifying themes relating movement in biological systems and machines. An important insight in this direction comes from the natural groupings of degrees of freedom and time scales in biological and engineering systems. Such grouping and separation can be treated from a geometric viewpoint using the formalisms and methods of differential geometry, Lie groups, and fiber bundles. Coupled oscillators provide the means to bind degrees of freedom either directly through phase locking or indirectly through geometric phases. This point of view leads to fresh ways of organizing the control structures of complex technological systems.

1. Introduction

In optics, lithography applications in micro-electronics, and in a variety of other contexts, the need for high-resolution motion control with high accuracy is met by specialized actuators that are quite different in their principles of operation from every-day devices such as electro-magnetic motors. One such device, manufactured by Burleigh Instruments under the trademark INCHWORM$^{\mathrm{TM}}$ is illustrated in Figure 1. This actuator, consisting of three sleeves/tubes made from piezo-electric material, mounted on a frame and enclosing a linear armature, works on the physical principle that the piezo-electric material deforms under electrical stimulus (the outer sleeves independently clamp down, and the middle sleeve stretches in length). Running the actuator through a succession of clamp-stretch-unclamp-unstretch cycles, one generates incremental motion of the armature in a specified direction. It is possible to make linear movements as small as 4 nanometers. Other actuators based on piezo-electric effects are increasingly finding their way into consumer products, including ultrasonic motors for auto-focussing in cameras based on surface wave

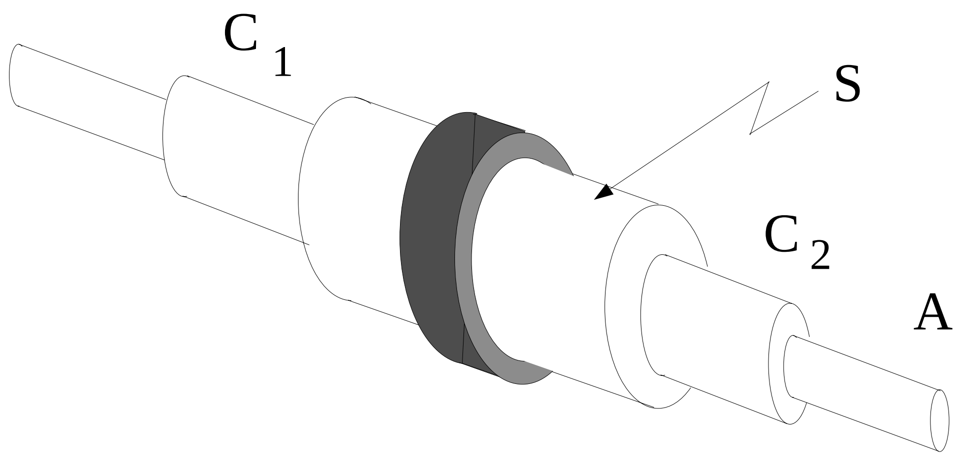

Figure 1. INCHWORM$^{\text{TM}}$
clamps C_1C_2, stretcher S, and armature A

excitation (see [31] for detailed discussions of these devices). A common design principle in these devices is a type of *rectification* of small cyclical motions to produce gross motions.

Turning to the natural world, much attention has been devoted to the systematic understanding of how various microscopic organisms move in fluids under various conditions. Such movement being essential to reaching food particles, efficiency considerations have also been of interest (see [7] for related discussion). Apparently, c.f. Figure 2, the paramecium gets around in a fluid under conditions of very low Reynolds number through a process of cyclical change in its boundary contour (or more precisely the envelope determined by the oscillating cilia that make up the contour). In the work of Shapere and Wilczek [28], under appropriate fluid mechanical assumptions, this has been shown to be the case via the mathematics of gauge theory (which has played an important role elsewhere in modern physics and geometry over the last three decades). Here again a type of rectification is at work.

While the low Reynolds number regime permits an essentially kinematic treatment of the paramecium, in other contexts of animal movement, dynamic influences play an important part (e.g. in walking, trotting and galloping gaits of quadrepeds c.f.[1], the swimming movement of the *lamprey* [4], etc.). A rather striking illustration of this occurs in the *Basiliscus plumifrons*, a type of lizard found in Central America that is capable of engaging in short bursts (range less than 10 meters) of walking on water, supporting itself through rapid pushing down (at 30 Hz frequency) by hind feet on the water. The reaction forces so generated are sufficient for support (see [18]).

Our examples are meant to underscore the principle of movement generation by repetitive, cyclical variation in certain degrees of freedom (of a machine or an organism) while constrained by interactions with the environment (e.g. ground contact, friction). Understanding this principle has had an important influence in recent research in control theory and in robotics, as also explained by the companion papers of R.W. Brockett, J.E. Marsden and R.M. Murray in this volume. Turning this principle into a quantitative tool requires an understanding of the rectification mechanism alluded to above. It is precisely

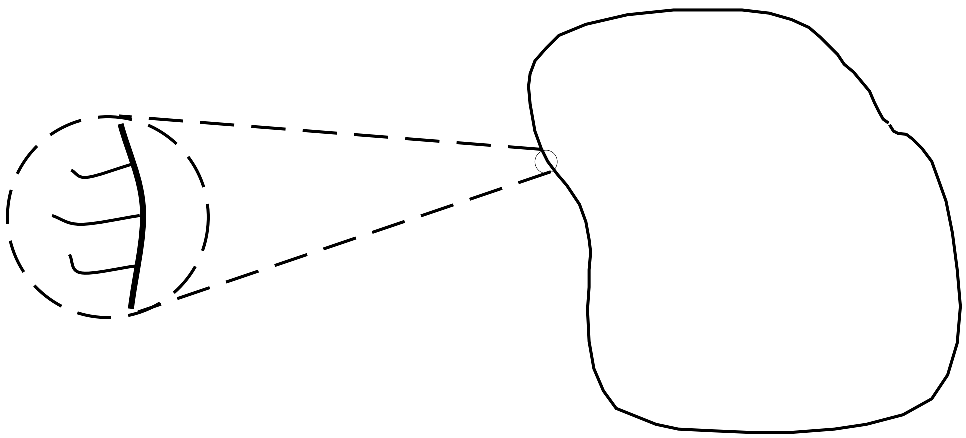

Figure 2. Paramecium

this mechanism, variously associated with geometric phases, area rules, and Lie bracket generation that has had a crucial role as a tool for designing machines, and algorithms to control them. In the context of motion generation, Brockett's paper [5] appears to be the first to state clearly and prove a version of the rule. See also [25] [26].

Placing the rectification principle in the broader context of motion control architectures for systems with many degrees of freedom is one of the goals of this paper. To clarify matters further, a toy example is given in Section 2, involving a unicycle and an oscillator.

There is already a rich tradition in biology and neuroscience of modeling movement via coupled oscillators. It is noteworthy that even in the presence of large numbers of degrees of freedom, motion control problems are effectively solved in biological systems of extraordinary variety. The work of Graham Brown [6] on half-centers and the fundamental investigations, starting in the 1930's, of N.A. Bernstein [2], on strategies for motion control continue to have influence in modern work (see Pearson's recent survey for a modern perspective [27]). Bernstein clearly identified a role for planning (i.e. feedforward control) along with feedback, regulation and tracking, in motion control. In Bernstein's scheme, adaptive restructuring of motion programs on-the-fly, through the use of afferent feedback pathways from mechano-receptors and other sensory modalities, had a prominent place. More recent work of neurophysiologists has focussed attention on *central pattern generators* (CPG's) in the nervous system as key to understanding the control of movement and posture (see [8-10], [13][14]). As mathematical objects, CPG's are networks of coupled oscillators and can be incorporated in the control architecture of a complex machine. Thus, if the state variables of the nodes of a CPG are in turn coupled to the degrees of freedom of the system to be controlled, it is possible to achieve coordination of the latter by prescribed phase coherence of the oscillators. The system to be controlled may be a multi-legged walking machine or a multi-fingered, anthropomorphic mechanical hand with built-in tactile sensors on the fingers. Sensory feedback paths to correct CPG dynamics would be necessary to provide a level of robustness to changes in the environment (e.g. obstacles, failures). These elements lead us to the architecture of Figure 4, discussed further in Section 3 of the paper.

In Section 4, we present a unifying geometric-mechanical picture of the ideas on rectification. The language of principal bundles, and connections goes hand-in-hand with the mechanical notions of configuration spaces, symmetries and constraints. Complementing the perspectives in the companion papers in this volume, we focus attention on the notion of averaging in Lie groups and it relation to rectification. In Section 5, we discuss some novel machines that illustrate the main ideas of this paper and point the way to further extensions.

2. From Shape Change to Global Movement

Our purpose here is to show how suitable notions of shape, together with cyclical shape change can yield global movement. In the case of the INCHWORM^{TM} actuator, the concept of shape can be identified with two pieces of information: the continuous elongation/contraction of the middle sleeve and the *discrete* state of the clamp-pair (which one is on or off?). For each such "shape", there is an associated holonomic constraint, and coordinated shape change together with switching of constraints leads to rectification and the travel of the armature. In a setting more natural for geometric arguments, piecewise holonomic constraints may be replaced by nonholonomic constraints, and this is best illustrated by classical mechanical examples involving the constraint of *no sliding* of a knife edge or *rolling without slipping* of a wheel on a surface. Consider for instance a unicycle with rider as in Figure 3. The kinematic equations of the unicycle are given by,

$$\begin{aligned} \dot{x} &= \cos(\phi) u_2 \\ \dot{y} &= \sin(\phi) u_2 \\ \dot{\phi} &= u_1. \end{aligned} \tag{1}$$

Here x and y denote the position of a fixed reference point on the unicycle and ϕ denotes the orientation of the unicycle relative to a fixed laboratory/observer frame. Further, u_1 denotes the steering speed, and u_2 denotes the heading speed, and these are assumed to be controllable by the unicyclist. From Eqn 1, it is clear that the constraint of no sliding,

$$-\dot{x}\sin(\phi) + \dot{y}\cos(\phi) = 0, \tag{2}$$

is maintained at all time.

Equations (1) and (2) can also be recast in the following equivalent form,

$$\dot{g} = g \cdot (A_1 u_1 + A_2 u_2) \tag{3}$$

where

$$g = \begin{pmatrix} \cos(\phi) & -\sin(\phi) & x \\ \sin(\phi) & \cos(\phi) & y \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

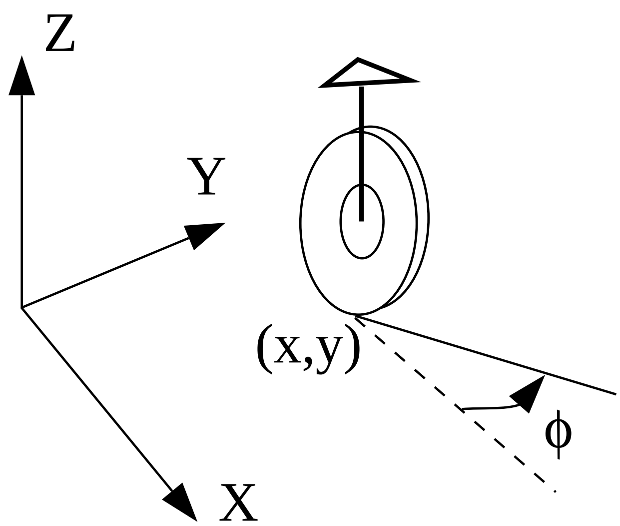

Figure 3. Unicycle

evolves in the group of rigid motions in the plane, with

$$
\begin{aligned}
A_1 &= \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
A_2 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\end{aligned} \tag{5}
$$

Imagine a typical unicyclist implementing pedaling and steering maneuvers that give rise to $u_i(t) = \epsilon f_i(t)$ where $f_i(.)$ are zero–mean periodic functions of time with a common period T, and $\epsilon > 0$ is a small amplitude parameter. In this instance, the "shape variables" $\tilde{u}_i(t) = \int_0^t u_i(\sigma) d\sigma$ are also periodic/oscillatory. Where does the unicycle end up? To get a decent approximation to the exact solution, one resorts to averaging theory c.f. [19][21]. In fact $g(t)$ is approximated up to quadratic terms in ϵ by the formula

$$g^{(2)}(t) = g(0) \cdot \exp\left(\sum_{i=1}^{3} z_i^{(2)}(t) A_i \right), \tag{6}$$

where, for $i = 1, 2$,

$$z_i^{(2)}(t) = \tilde{u}_i(t) + z_{i0}^{(2)} \tag{7}$$

and

$$z_3^{(2)}(t) = \frac{t}{T} \text{Area}_{1,2}(T) + z_{30}^{(2)}. \tag{8}$$

Here, the $z_{i0}^{(2)}$ are initial conditions, the matrix

$$A_3 = A_1 A_2 - A_2 A_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

is the Lie bracket of A_1 and A_2 and,

$$\text{Area}_{1,2}(T) = \frac{1}{2}\int_0^T \big(\tilde{u}_1(\sigma)\dot{\tilde{u}}_2(\sigma) - \tilde{u}_2(\sigma)\dot{\tilde{u}}_1(\sigma)\big)d\sigma, \tag{9}$$

is the area of the loop in shape space executed by the unicyclist in the course of the chosen oscillatory maneuver.

Formula (6) predicts a secular drift in the direction of the Lie bracket A_3, and following (8) and (9), illustrates the rectification principle as an *area–rule.* By a succession of oscillatory maneuvers, the unicyclist can get around anywhere and manage parallel parking! (For related ideas and references, see the paper of Murray in this volume.) This hinges on the fact that the constraint (2) is nonholonomic or equivalently, the Lie bracket A_3 is linearly independent of A_1 and A_2, the directions trivially controllable by the unicyclist. It is however important that the phase relations between the pedaling oscillations and the steering oscillations be right, or else the area $\text{Area}_{1,2}(T)$ will vanish, killing the secular drift. This brings up the need for coupled oscillations. Appropriate shape variations may be drawn from solutions to variational problems.

The unicycle example illustrates a geometric interpretation of shape and shape change. For a six–legged insect (or machine) with legs capable of lift and swing, the shape space may be a submanifold of a 6×3 –dimensional torus. Shape change in that case is achieved via successively lifting and swinging the legs before returning to ground contact.

3. Scripts and Oscillators

Area rules of the type discussed in the last section have been used in developing computer programs to synthesize feedforward control laws (motion scripts), in [19][21]. We think of such programs as *script generators*, producing detailed streams of instructions to machines. One such implementation is used to control an underwater vehicle [20][21]. Integrating such script generators into a larger framework for *intelligent control* of movement is a major challenge (–the framework has to accommodate uncertainty, limited sensing of the environment, obstacles that move about, rough terrain etc.) and we argue that there is much insight to be gained from deeper study of biological motion control systems.

The Russian physiologist N. Bernstein, in his studies of the movement problem, proposed a variety of architectural principles. Given the large numbers of degrees of freedom involved in even elementary motor acts, binding (or synchronization of) the degrees of freedom into groups is necessary. Such binding has to be dynamic to accommodate varying stages in a movement. Bernstein viewed rythm generators or oscillators as the means to implement binding. Bernstein also viewed as central to motor control, the ability to change a motor program in the middle of a movement, possibly based on data from afferent sensory pathways. Much work, since Graham Brown's proposal of half-centers has gone into understanding how neural circuitry could be organized to produce temporal patterns of neuronal firings that seem to be responsible for rythmic movements. See for instance the compendium of papers [10]. The oscillations in the temporal patterns are assumed to be in correspondence with actual movements produced, for instance, a particular *gait,* i.e. rythmic stepping, in a quadraped. A complex movement could be segmented into distinctive gaits, and modules capable of piecing together such segments prior to initiation of a movement and altering them 'on-the-fly' are essential to intelligent control. Further it is plausible that in biological systems, the higher cognitive elements engaged in movement control primarily pay attention to a symbolic description of movement ignoring detailed timing information. For instance, in the case of a six-legged insect, identifying the legs on the left (respectively right) side of the body with the symbols L_i (respectively R_i), where the index i runs from 1 to 3, (3 stands for the hind legs, 2 for the middle legs, and 1 for the front legs), one can refer to a gait pattern by a string of symbols, as in,

$$(a) \quad R_3, R_2, R_1, L_3, L_2, L_1$$

$$(b) \quad L_2, R_3L_1, R_2, R_1L_3$$

$$(c) \quad R_3L_2R_1, L_3R_2L_1$$

These strings are to be interpreted as defining the sequence in which each leg is lifted from the ground, and symbols in a group *not* separated by a comma correspond to synchronized leg-lifts. Thus string (c) above represents the so-called alternating tripod gait, being the fastest, and the string (a) above stands for the slowest. Both timing and step length information are hidden, although it is experimentally observed that swing times are independent of gait. A suitable control framework would need to be able to accommodate descriptions of movement both at the symbolic and at the detailed timing level. In fact one can even work out an admissible language for movements by stringing together words as in (a) (b) (c).

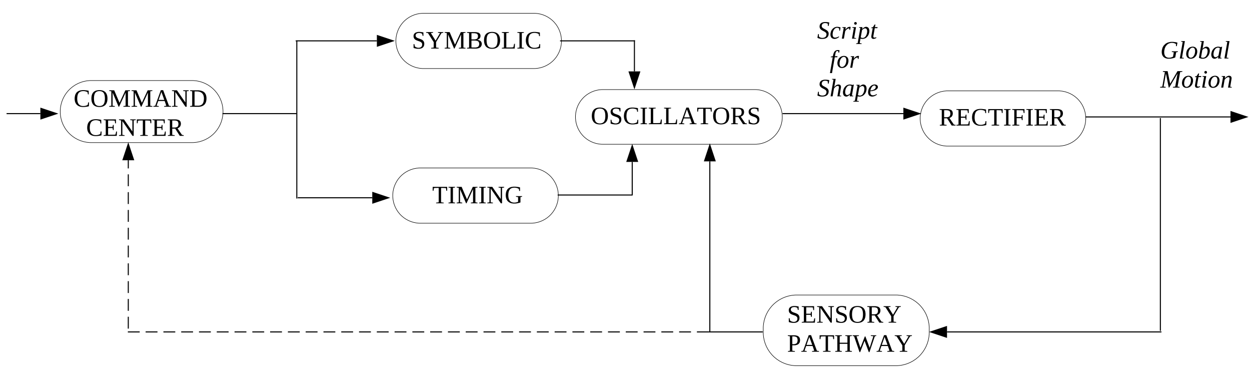

Figure 4. Architecture for Motion Control

Based on these insights from biology, one is led to a possible architecture for intelligent control of movement as in Figure 4. Here the command center communicates a prescribed global movement command to be transformed into symbolic and timing instructions which are then implemented by a network of coupled oscillators that produce the script for shape change. The rectification path produces the desired global movement. Sensory information is returned to the command center and possibly to the oscillators to modify/correct the motion commands and scripts. In practice, in legged animals or machines, this afferent pathway may lead to script change (gait switching). For a robust, model-independent approach to gait switching on the basis of bifurcation theory in the presence of symmetries, see [11]. In our own work, the change in control authority that accompanies the failure of actuators is one of the sources of script change [20].

The architecture sketched out here give prominence to what may be a missing ingredient in much of the discussion of rythmic movements in biology, –namely the rectification module. In his paper in the present volume Brockett takes the view that rectification is a tool for 'approximate inversion' of motion specifications over time and shows how oscillators do the job. It may be possible to suggest some biological experiments to determine if indeed such approximate inverses are learned from repeated trials, lessening thereby the need to store motion scripts. Finally, it should be added that there are software aspects to the control architecture we discussed that makes contact with current thinking in re-configurable software for robotics, c.f. the work on CHIMERA [29].

We close this section by pointing out exciting new developments in the direction of incorporating a combination of pattern generating oscillators, elasto-mechanics based models of body movement and muscle response, and models of fluid interaction with the skin to capture the complexities of lamprey movement [4]. At least in one machine that we have studied (see Section 5), several of these ingredients prove to be necessary for complete understanding.

4. Unifying Geometry

The model (3) of the unicycle is not so special as it might seem at first glance. In practice, the models of mechanics governing the behavior of a wide variety of machines, admit certain unifying geometric elements. The possible system configurations constitute the space Q. There is always the symmetry of Newtonian mechanics, namely indifference of the dynamics to change of inertial observer. More generally, one has a Lie group G of invertible transformations acting on Q that leave the Lagrangian of the system invariant and possibly any applicable constraints as well. The equivalence classes defined by the orbits of G can be brought to one-to-one correspondence with the space $S = Q/G$ of shapes. The triple (Q, G, S) is known as a principal bundle. Most of the examples one encounters in mechanical settings can be given the structure of a *trivial* principal bundle, i.e. the configuration space looks like a product $S \times G$, a simplification we shall assume from here on. Each configuration will then be a pair $q = (x, g)$.

If sufficiently many independent constraints (analogous to the constraint of no sliding in the unicycle example) are present then, it is possible to construct a well-defined splitting of the space of velocities (tangent bundle TQ of the configuration space) at every configuration, into a set of symmetry directions along group orbits (also called vertical directions) and a set of complementary directions, called horizontal directions, *isomorphic* to the space of directions along which one can change the shape. One then says that the bundle (Q, G, S) has acquired a *principal connection*. (See Figure 5 for a sketch of the geometric set-up). The curvature of the connection has a great to deal to say about the following question. In analogy with the unicyclist's problem, where do we end up in the

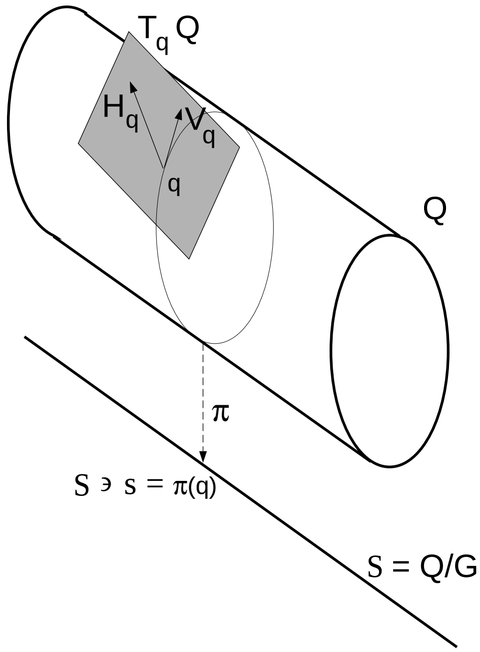

Figure 5. Principal Bundle with Connection

configuration space Q when we make a cyclic movement in the space of shapes? The constraints are sufficient to determine a relation between the evolution of $g(t)$ and the shape trajectory $x(t)$, of the form,

$$\dot{g} = -g \cdot \xi(x, \dot{x}) \tag{10}$$

where ξ represents the *connection form* and is linear in $\dot{x}$. Despite the complication arising from the connection form, Eqn. 10 is a good deal like Eqn. 3. The concept of holonomy in differential geometry gives a formal answer to the above question. Drift in the group variables can be built up by repetitively traversing the same loop in shape space. If the shape velocity is of the form $\dot{x} = \epsilon \cdot f(t)$, where ϵ is a small amplitude parameter then as in Section 2, one can give an approximate solution to Eqn. 10 using the theory of averaging. This is done in [19,21], leading to area-rules. Once again the area-rules yield constructive procedures for generating suitable movements in shape space to achieve required transport in configuration space.

The unifying geometric point of view of this section is very useful in formulating answers to constructive controllability questions arising in, the study of maneuvers of space-based robotic devices [15][32], the problem of the paramecium at low Reynolds number studied by Shapere and Wilczek [28], and a wide variety of nonholonomically constrained problems. There is much that needs to be done to integrate this geometric viewpoint into the control architecture presented in Section 3. In particular, the capability to adapt motion scripts in this level of generality, based on sensory inputs, probably needs new mathematical apparatus.

5. Some Interesting Machines

Some of the ideas we presented here have been tested in the laboratory and in simulation. In the thesis of Manikonda [22], motion planning for nonholonomic robots in the presence of obstacles is investigated from a perspective close to the one we discuss. Over the years, there has been growing interest in robotic machines that exploit principles of movement found in biological systems. The excellent book of Hirose [12] contains many examples of successful designs and algorithms. Encouraged by certain designs for redundant manipulators developed by Joel Burdick and his students at Caltech, we investigated a variety of machines that could be controlled via shape change. One such instance is the nonholonomic variable geometry truss (NVGT) in Figure 6. This machine consists of a pair of modules on idler wheels, rolling without slipping on a surface, with deformable bodies. The intent is to drive this machine entirely by deformations of the body using the connecting links, without any direct actuation of the wheels.

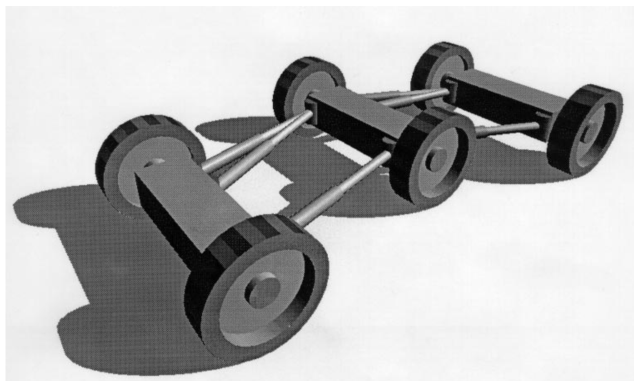

Figure 6. Two Module Nonholonomic Variable Geometry Truss

The NVGT fits nicely into the framework of this paper. The configuration space is the cartesian product of 3 copies of the rigid motion group $SE(2)$, and the symmetry group is also $SE(2)$. Thus the shape space is $S = SE(2) \times SE(2)$, representing the freedom to alter the shape by changing the lengths of the connecting links in each module. Away from certain singular configurations, determined to be those for which all three axles intersect (possibly at infinity) , the geometry of Section 4 applies and the 'no sliding' constraints fix a principal connection. Cyclical shape changes produce, snake-like movement of the machine, c.f. [16][17][30].

It should be clear that additional modules could be attached to the NVGT of Figure 6, thereby increasing the number of constraints and the number of degrees of freedom. In that case, as shown in [17][30], the problem becomes over-constrained thus limiting the allowable shape changes. This in itself is not a disadvantage in selecting shape change scripts.

Suppose now that one of the modules in Figure 6 is detached and we are left with just one module. In this case the problem is under-constrained, and one does not quite have the geometry of Section 4. One does not obtain a principal connection from the 'no sliding' constraints alone. There is a subtler symmetry in the problem, that arises from the interaction between the original Newtonian symmetry and the constraints, which yields a new momentum equation that the trajectories of the system must obey. The

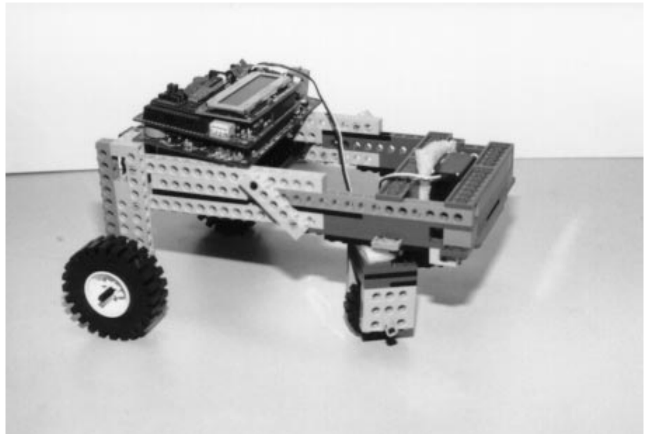

Figure 7. Computer Controlled Roller Racer

main ideas behind this new symmetry have only recently become clear in the work of [3]. To illustrate this, a machine modeled on the patented toy Roller Racer (U.S. patent #3663038 of May 16, 1972), c.f. Figure 7 was built. This device is a special case of the single module nonholonomic variable geometry truss on wheels, with only a single degree of shape freedom. The shape space in this case is the circle S^1. It is remarkable, that in this case the full theory of nonholonomic momentum equation applies and using this extra equation, one formulates a principal connection on the bundle $(S^1 \times SE(2), SE(2), S^1)$. Motion control by periodic forcing of one degree of shape freedom is accomplished. Details are to be found in [30].

The last-mentioned example uses dynamical information in an essential way and in some sense there are parallels between this investigation and the work of [4] on the lamprey. The rich variety of global motions can be best understood by the proper synthesis of kinematic, geometric and dynamic information and the principle of rectification applied to cyclical shape variations is an effective guide even in this mathematically complex setting. An intelligent control architecture based on such synthesis would be of great interest. Again nature would have taught us to build better machines.

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